positive, then the distribution is more peaked than a normal distribution. A negative value means that the distribution is flatter than a normal distribution. The value of this statistic for Table 15.2 is –1.261, indicating that the distribution is flatter than a normal distribution. Measures of shape are important, because if a distribution is highly skewed or markedly peaked or flat, then statistical procedures that assume normality should be used with caution.

ACTIVE RESEARCH

Wendy's Customers: Who Are the Heavyweights?

Visit www.wendys.com and conduct an Internet search using a search engine and your library's online database to obtain information on the heavy users of fast-food restaurants.

As the marketing director for Wendy's, how would you target the heavy users of fast-food restaurants?

In a survey for Wendy's, information was obtained on the number of visits to Wendy's per month. How would you identify the heavy users of Wendy's and what statistics would you compute to summarize the number of visits to Wendy's per month?

Introduction to Hypothesis Testing

Basic analysis invariably involves some hypothesis testing. Examples of hypotheses generated in marketing research abound:

- The department store is being patronized by more than 10 percent of the households.
- The heavy and light users of a brand differ in terms of psychographic characteristics.
- One hotel has a more upscale image than its close competitor.
- Familiarity with a restaurant results in greater preference for that restaurant.

Chapter 12 covered the concepts of the sampling distribution, standard error of the mean or the proportion, and the confidence interval.[5] All these concepts are relevant to hypothesis testing and should be reviewed. Now we describe a general procedure for hypothesis testing that can be applied to test hypotheses about a wide range of parameters.

A General Procedure for Hypothesis Testing

The following steps are involved in hypothesis testing (Figure 15.3).

1. Formulate the null hypothesis H_0 and the alternative hypothesis H_1.
2. Select an appropriate statistical technique and the corresponding test statistic.
3. Choose the level of significance, α.
4. Determine the sample size and collect the data. Calculate the value of the test statistic.
5. Determine the probability associated with the test statistic under the null hypothesis, using the sampling distribution of the test statistic. Alternatively, determine the critical values associated with the test statistic that divide the rejection and nonrejection regions.
6. Compare the probability associated with the test statistic with the level of significance specified. Alternatively, determine whether the test statistic has fallen into the rejection or the nonrejection region.
7. Make the statistical decision to reject or not reject the null hypothesis.
8. Express the statistical decision in terms of the marketing research problem.

null hypothesis
A statement in which no difference or effect is expected. If the null hypothesis is not rejected, no changes will be made.

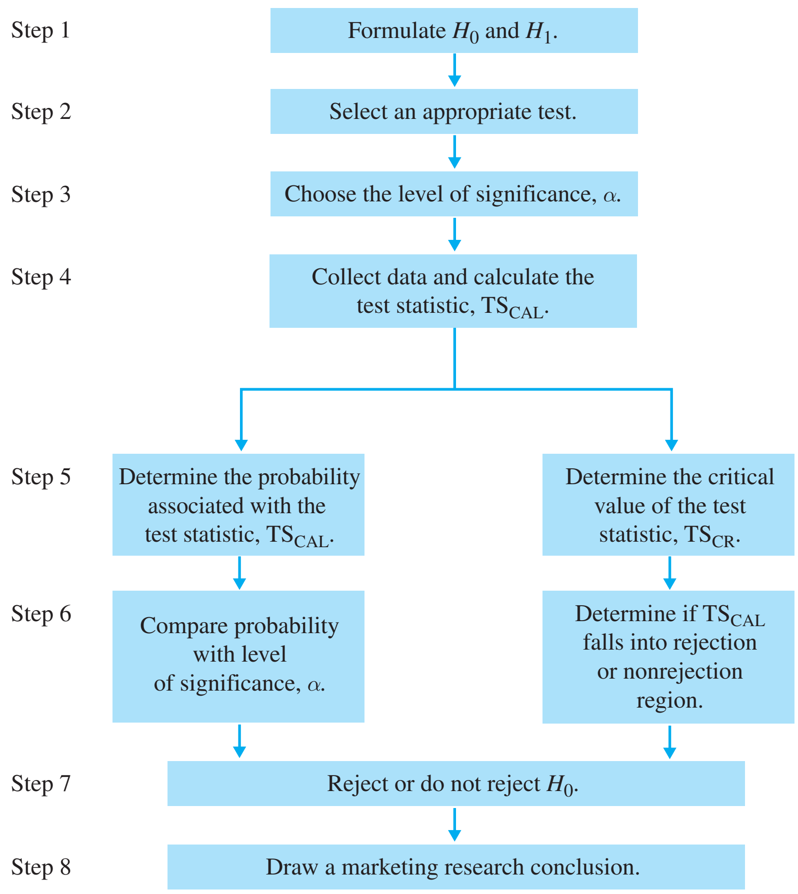
alternative hypothesis
A statement that some difference or effect is expected. Accepting the alternative hypothesis will lead to changes in opinions or actions.

Step 1: Formulate the Hypotheses

The first step is to formulate the null and alternative hypotheses. A **null hypothesis** is a statement of the status quo, one of no difference or no effect. If the null hypothesis is not rejected, no changes will be made. An **alternative hypothesis** is one in which some difference or effect is expected. Accepting the alternative hypothesis will lead to changes in opinions or actions. Thus, the alternative hypothesis is the opposite of the null hypothesis.

The null hypothesis is always the hypothesis that is tested. The null hypothesis refers to a specified value of the population parameter (e.g., μ, σ, π), not a sample statistic (e.g., $\overline{X}$, s, p). A null hypothesis may be rejected, but it can never be accepted based on a single test. A statistical test can have one of two outcomes. One is that the null hypothesis is rejected and the alternative

FIGURE 15.3
A General Procedure for Hypothesis Testing



hypothesis is accepted. The other outcome is that the null hypothesis is not rejected based on the evidence. However, it would be incorrect to conclude that because the null hypothesis is not rejected, it can be accepted as valid. In classical hypothesis testing, there is no way to determine whether the null hypothesis is true.

In marketing research, the null hypothesis is formulated in such a way that its rejection leads to the acceptance of the desired conclusion. The alternative hypothesis represents the conclusion for which evidence is sought. For example, a major department store is considering the introduction of an Internet shopping service. The new service will be introduced if more than 40 percent of the Internet users shop via the Internet. The appropriate way to formulate the hypotheses is:

$$H_0: \pi \leq 0.40$$
$$H_1: \pi > 0.40$$

If the null hypothesis H_0 is rejected, then the alternative hypothesis H_1 will be accepted and the new Internet shopping service will be introduced. On the other hand, if H_0 is not rejected, then the new service should not be introduced unless additional evidence is obtained.

one-tailed test
A test of the null hypothesis where the alternative hypothesis is expressed directionally.

two-tailed test
A test of the null hypothesis where the alternative hypothesis is not expressed directionally.

This test of the null hypothesis is a **one-tailed test**, because the alternative hypothesis is expressed directionally: The proportion of Internet users who use the Internet for shopping is greater than 0.40. On the other hand, suppose the researcher wanted to determine whether the proportion of Internet users who shop via the Internet is different from 40 percent. Then a **two-tailed test** would be required, and the hypotheses would be expressed as:

$$H_0: \pi = 0.40$$
$$H_1: \pi \neq 0.40$$

In commercial marketing research, the one-tailed test is used more often than a two-tailed test. Typically, there is some preferred direction for the conclusion for which evidence is sought. For example, the higher the profits, sales, and product quality, the better. The one-tailed test is more powerful than the two-tailed test. The power of a statistical test is discussed further in step 3.

Step 2: Select an Appropriate Test

test statistic
A measure of how close the sample has come to the null hypothesis. It often follows a well-known distribution, such as the normal, *t*, or chi-square distribution.

To test the null hypothesis, it is necessary to select an appropriate statistical technique. The researcher should take into consideration how the test statistic is computed and the sampling distribution that the sample statistic (e.g., the mean) follows. The **test statistic** measures how close the sample has come to the null hypothesis. The test statistic often follows a well-known distribution, such as the normal, *t*, or chi-square distribution. Guidelines for selecting an appropriate test or statistical technique are discussed later in this chapter. In our example, the z statistic, which follows the standard normal distribution, would be appropriate. This statistic would be computed as follows:

$$z = \frac{p - \pi}{\sigma_p}$$

where

$$\sigma_p = \sqrt{\frac{\pi(1 - \pi)}{n}}$$

Step 3: Choose Level of Significance, α

Whenever we draw inferences about a population, there is a risk that an incorrect conclusion will be reached. Two types of errors can occur.

Type I error
Also known as *alpha error*, it occurs when the sample results lead to the rejection of a null hypothesis that is in fact true.

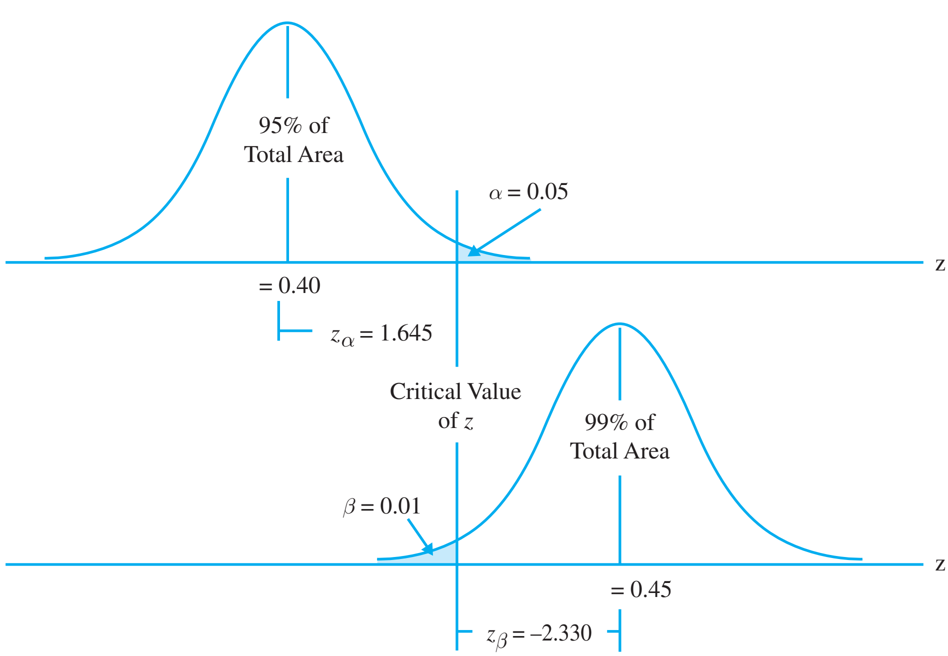
level of significance
The probability of making a Type I error.

TYPE I ERROR **Type I error** occurs when the sample results lead to the rejection of the null hypothesis when it is in fact true. In our example, a Type I error would occur if we concluded, based on the sample data, that the proportion of customers preferring the new service plan was greater than 0.40, when in fact it was less than or equal to 0.40. The probability of Type I error (α) is also called the **level of significance**. The Type I error is controlled by establishing the tolerable level of risk of rejecting a true null hypothesis. The selection of a particular risk level should depend on the cost of making a Type I error.

Type II error
Also known as *beta error*, it occurs when the sample results lead to the nonrejection of a null hypothesis that is in fact false.

TYPE II ERROR **Type II error** occurs when, based on the sample results, the null hypothesis is not rejected when it is in fact false. In our example, the Type II error would occur if we concluded, based on sample data, that the proportion of customers preferring the new service plan was less than or equal to 0.40 when, in fact, it was greater than 0.40. The probability of Type II error is denoted by β. Unlike α, which is specified by the researcher, the magnitude of β depends on the actual value of the population parameter (proportion). The probability of Type I error (α) and the probability of Type II error (β) are shown in Figure 15.4. The complement $(1 - \beta)$ of the probability of a Type II error is called the *power of a statistical test.*

FIGURE 15.4
Type I Error (α) and Type II Error (β)



power of a test
The probability of rejecting the null hypothesis when it is in fact false and should be rejected.

POWER OF A TEST The **power of a test** is the probability $(1 - \beta)$ of rejecting the null hypothesis when it is false and should be rejected. Although β is unknown, it is related to α. An extremely low value of α (e.g., $= 0.001$) will result in intolerably high β errors. So it is necessary to balance the two types of errors. As a compromise, α is often set at 0.05; sometimes it is 0.01; other values of α are rare. The level of α along with the sample size will determine the level of β for a particular research design. The risk of both α and β can be controlled by increasing the sample size. For a given level of α, increasing the sample size will decrease β, thereby increasing the power of the test.

Step 4: Collect Data and Calculate Test Statistic

Sample size is determined after taking into account the desired α and β errors and other qualitative considerations, such as budget constraints. Then the required data are collected and the value of the test statistic computed. In our example, 30 users were surveyed and 17 indicated that they used the Internet for shopping. Thus the value of the sample proportion is $p = 17/30 = 0.567$.

The value of σ_p can be determined as follows:

$$\sigma_p = \sqrt{\frac{\pi(1 - \pi)}{n}}$$

$$= \sqrt{\frac{(0.40)(0.60)}{30}}$$

$$= 0.089$$

The test statistic z can be calculated as follows:

$$z = \frac{p - \pi}{\sigma_p}$$

$$= \frac{0.567 - 0.40}{0.089}$$

$$= 1.88$$

Step 5: Determine the Probability (or Critical Value)

Using standard normal tables (Table 2 of the Statistical Appendix), the probability of obtaining a z value of 1.88 can be calculated (see Figure 15.5). The shaded area between $-\infty$ and 1.88 is 0.9699. Therefore, the area to the right of $z = 1.88$ is $1.0000 - 0.9699 = 0.0301$. This is also called the ***p* value** and is the probability of observing a value of the test statistic as extreme as, or more extreme than, the value actually observed, assuming that the null hypothesis is true.

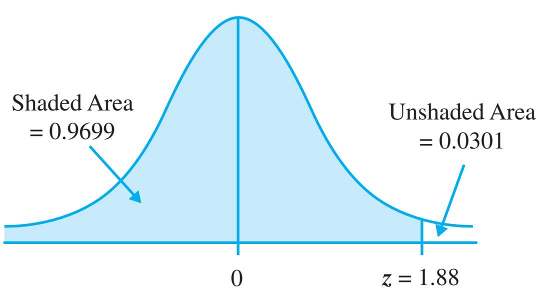
***p* value**
This is the probability of observing a value of the test statistic as extreme as, or more extreme than, the value actually observed, assuming that the null hypothesis is true.

Alternatively, the critical value of z, which will give an area to the right side of the critical value of 0.05, is between 1.64 and 1.65 and equals 1.645. Note that in determining the critical value of the test statistic, the area in the tail beyond the critical value is either α or $\alpha/2$. It is α for a one-tailed test and $\alpha/2$ for a two-tailed test.

Steps 6 and 7: Compare the Probability (or Critical Value) and Make the Decision

The probability associated with the calculated or observed value of the test statistic is 0.0301. This is the probability of getting a p value of 0.567 when $\pi = 0.40$. This is less than the level of

FIGURE 15.5
Probability of *z* with a One-Tailed Test



significance of 0.05. Hence, the null hypothesis is rejected. Alternatively, the calculated value of the test statistic $z = 1.88$ lies in the rejection region, beyond the value of 1.645. Again, the same conclusion to reject the null hypothesis is reached. Note that the two ways of testing the null hypothesis are equivalent but mathematically opposite in the direction of comparison. If the probability associated with the calculated or observed value of the test statistic (TS_{CAL}) is *less than* the level of significance (α), the null hypothesis is rejected. However, if the absolute value of the calculated value of the test statistic is *greater than* the absolute value of the critical value of the test statistic (TS_{CR}), the null hypothesis is rejected. The reason for this sign shift is that the larger the absolute value of TS_{CAL}, the smaller the probability of obtaining a more extreme value of the test statistic under the null hypothesis. This sign shift can be easily seen:

if probability of $TS_{CAL} <$ significance level (α), then reject H_0,

but

if $|TS_{CAL}| > |TS_{CR}|$, then reject H_0.

Step 8: Marketing Research Conclusion

The conclusion reached by hypothesis testing must be expressed in terms of the marketing research problem. In our example, we conclude that there is evidence that the proportion of Internet users who shop via the Internet is significantly greater than 0.40. Hence, the recommendation to the department store would be to introduce the new Internet shopping service.

As can be seen from Figure 15.6, hypotheses testing can be related to either an examination of associations or an examination of differences. In tests of associations, the null hypothesis is that there is no association between the variables (H_0: . . . is NOT related to . . .). In tests of differences, the null hypothesis is that there is no difference (H_0: . . . is NOT different from . . .). Tests of differences could relate to distributions, means, proportions, medians, or rankings. First, we discuss hypotheses related to associations in the context of cross-tabulations.

Cross-Tabulations

Although answers to questions related to a single variable are interesting, they often raise additional questions about how to link that variable to other variables. To introduce the frequency distribution, we posed several representative marketing research questions. For each of these, a researcher might pose additional questions to relate these variables to other variables. For example:

- How many brand-loyal users are males?
- Is product use (measured in terms of heavy users, medium users, light users, and nonusers) related to interest in outdoor activities (high, medium, and low)?
- Is familiarity with a new product related to age and education levels?
- Is product ownership related to income (high, medium, and low)?

The answers to such questions can be determined by examining cross-tabulations. Whereas a frequency distribution describes one variable at a time, a **cross-tabulation** describes two or

cross-tabulation
A statistical technique that describes two or more variables simultaneously and results in tables that reflect the joint distribution of two or more variables that have a limited number of categories or distinct values.

FIGURE 15.6
A Broad Classification of Hypothesis Tests