UNIT-1

Quantam mechanics

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* wave particle dualism- As light dual nature in centain phenomena light behave like a wave such as interfarance deflection in contain other phenomena it behave like a particle such as photo electric effect and compton effect.

* De- Broglie interduce that matter also should have dual nature. The wave length of that wave is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where v = velocity of particle
h = plank's constant = 6.63×10^{-34}
m = mass of particle

Imp De-Broglie Hypothesis- was Based upon the following effects.

(i) Light and a matter both are the form of energy and they can be interchange.

(ii) As Light has dual nature so, matter also should have dual nature.

"According to De-Broglie Hypothesis - a wave is associated with moving particle".

The wave length of the wave associated with moving particle of mass (m) moving with velocity (v) as given by-

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

Expression for De-Broglie wave length-

Imp

According to Einstein's mass energy relation the energy is defined as.

$$E = mc^2 \quad -①$$

According to Energy of Photon is also defined as

$$E = h\nu$$

$$E = \frac{hc}{\lambda} \quad -② \qquad \{c = \nu\lambda\}$$

From eq① and eq② we have

$$\frac{hc}{\lambda} = mc^2$$

$$mc = \frac{h}{\lambda}$$

$$\boxed{\lambda = \frac{h}{mc}}$$

If the moving particle is meterial particle then $c \to v$

$$\boxed{\lambda = \frac{h}{mv}} \qquad \{P = mv\}$$

$$\boxed{\lambda = \frac{h}{P}}$$

Different Expression for De-Broglie wave length-

① ~~wavelength~~

① The K.E. of moving particle of mass (m) moving with velocity (v) is given by

$$E = \frac{1}{2}mv^2 \qquad \{K.E. = E\}$$

$$E = \frac{(mv)^2}{2m}$$

$$E = \frac{p^2}{2m}$$

$$p = \sqrt{2mE}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

② If the moving particle is a charged particle of charge (q) accelarated by a potential difference then

$$E = qV$$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2qVm}}$$

③ If the moving particle as n electron-

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}}$$

$$\lambda = \frac{12.28}{\sqrt{V}} \mathring{A}$$

④ If the moving particle is Neutron then

$$\lambda = \frac{h}{\sqrt{2me\left[\frac{E}{e}\right]}}$$

$$\lambda_n = \frac{h}{\sqrt{2me\,E_{eV}}}$$

$$\lambda_n = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} \times E_{eV}}}$$

$$\boxed{\lambda_n = \frac{0.286}{\sqrt{E_{eV}}}\ \text{Å}}$$

(5) If the particle is in thermal equilabrium then

$$E = \frac{3}{2}KT$$

then
$$\lambda = \frac{h}{\sqrt{2m \times \frac{3}{2}KT}}$$

$$\lambda = \frac{h}{\sqrt{3mKT}}$$

Que. $V = 100V$ $\quad {}_2He^4$

$$\alpha = {}_2He^4$$

$$\lambda_\alpha = \frac{h}{\sqrt{2mqV}}$$

$$\lambda_\alpha = \frac{6.63\times10^{-34}}{\sqrt{2\times4\times1.67\times10^{-27}\times2\times1.6\times10^{-19}\times200}}$$

$$\lambda_\alpha = 0.0017\ \text{Å}$$

* properties of De-Broglie matter wave- De-Broglie matter wave have the following characteristic-

→ Larger the velocity of the particle smaller the wave length of the wave associated with it and vice-versa.

→ when $\vartheta = 0$ then $\lambda = \infty$ that is the wave is associated only with moving particle.

→ Smaller the mass of particle, Larger the wavelength of the wave associated with it and vice-versa.

→ Both aspects never exist together in same experiment.

→ matter wave are not electromagnatic wave.

Expression for the velocity of De-Brogli matter wave-

The Energy of material particle can be express as-

(i) $E = mc^2$

The energy is also defined as-

$$E = h\vartheta$$

$$E = mc^2 = h\vartheta$$

$$\vartheta = \frac{mc^2}{h} \quad -(1)$$

The wavelength of De-Brogli matter wave is given by

$$\lambda = \frac{h}{m\vartheta} \quad -(2)$$

where, ϑ = velocity of particle

Imp

velocity of De-Brogli matter wave OR wave velocity OR (Phase velocity) -

Phase velocity is defined as

$$\boxed{v_p = \vartheta\lambda}$$

From eq(1) and eq(2)

$$v_p = \frac{mc^2}{h} \times \frac{h}{m\vartheta}$$

$$\boxed{v_p = \frac{c^2}{v}}$$

From here its clear that the phase velocity is grather than the speed of light.

Imp (ii) <u>wave velocity (v_p)</u>- (Phase velocity) Phase velocity is defined as the Ratio of angular velocity the per propergution constant.

$$\boxed{v_p = \frac{\omega}{k}}$$

(iii) equation of wave can be written as -

$$Y = A\sin(\omega t - kx)$$

$$\therefore \omega t - kx = \text{const.}$$

diff w.r. to t we have

$$\omega - k\frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = \frac{\omega}{k}$$

$$\boxed{v_p = \frac{\omega}{k}}$$

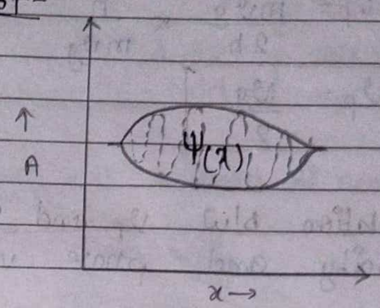
That is the velocity of constant phase of a wave is known phase velocity.

Imp <u>Group velocity</u>- when a group of wave propagate along the same direction then the observed velocity is the velocity of maximum amplitude.
which are known as Group velocity.

That is $\boxed{v_g = \frac{d\omega}{dk}}$

Thus, the group velocity is define as the ratio of change in angular velocity to the change in propagation constant.

<u>Wave packet</u>-



when a group of wave propagate along the same direction having amplitude and phase search that they interfair constructively orr a small region of space. where the particle can be located is known as wave packet.

The velocity of wave packet is also known as group velocity.

Relation b/w v_p and v_g for Non- Relativstics free particle

Energy of Non- Relativstics free particle can be express as.

$$E = h\nu = \frac{1}{2}mv_g^2$$

$$\boxed{\nu = \frac{mv_g^2}{2h}} \quad (1)$$

The wavelength is define as-

$$\boxed{\lambda = \frac{h}{mv_g}} \quad (2)$$

The wave velocity is given by -

$$\boxed{v_p = \nu \lambda}$$

then

$$v_p = \frac{m v_g^2}{2h} \times \frac{h}{m v_g}$$

$$\boxed{v_p = \frac{v_g}{2}}$$

Imp # General Relation b/w v_p and v_g → The group velocity and phase velocity are define as

$$\boxed{v_g = \frac{d\omega}{dk}} \quad -(1)$$

$$\boxed{v_p = \frac{\omega}{k}} \quad -(2)$$

from eq^n (1)

$$v_g = \frac{d\omega}{d\left(\frac{2\pi}{\lambda}\right)} \qquad \left\{ k = \frac{2\pi}{\lambda} \right\}$$

$$v_g = \frac{d\omega}{2\pi\left(-\lambda^{-2}\, d\lambda\right)}$$

$$\boxed{v_g = \frac{-\lambda^2}{2\pi} \frac{d\omega}{d\lambda}} \quad (3)$$

from eq^n (2) we have-

$$\omega = k \cdot v_p$$

$$\omega = 2\pi \frac{v_p}{\lambda}$$

diff w.r. to λ

$$\frac{d\omega}{d\lambda} = 2\pi \left[\frac{\lambda \frac{dv_p}{d\lambda} - v_p}{\lambda^2} \right] \quad \text{put in eq}^n \ (3)$$

$$v_g = \frac{-\lambda^2}{2\pi} \times 2\pi \left[\frac{\lambda \frac{dv_p}{d\lambda} - v_p}{\lambda^2} \right]$$

$$\boxed{v_g = v_p - \lambda \frac{dv_p}{d\lambda}}$$ This is relation b/w v_p and v_g.

Case-I If the medium is ~~known~~ of non-dispersive.

$$\frac{dv_p}{d\lambda} = 0$$

$$\boxed{v_p = v_g}$$

Imp
Que. show that
(i) $v_g = v$
(ii) $v_p v_g = c^2$

Sol^n - proof (i)
The group velocity can be expressed as-

$$v_g = \frac{d\omega}{dk}$$

$$\boxed{v_g = \frac{d\omega / dv}{dk / dv}} \quad (1)$$

The angular velocity is define as

$$\omega = 2\pi\nu = \frac{2\pi E}{h} = \frac{2\pi (mc^2)}{h}$$

$$\boxed{\omega = \frac{2\pi m_0 c^2}{h\left(1 - \frac{v^2}{c^2}\right)^{1/2}}} \quad -(2) \qquad \left\{ m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \right\}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\frac{h}{mv}} = \frac{2\pi m v}{h}$$

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$$\boxed{k = \frac{2\pi m_0 v}{h\left(1-\frac{v^2}{c^2}\right)^{1/2}}} \quad -(3)$$

The diff eq(2) w.r. to v

$$\frac{d\omega}{dv} = \frac{2\pi m_0 c^2}{h} \frac{d}{dv}\left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0 c^2}{h}\left[\frac{-1}{2}\left[1-\frac{v^2}{c^2}\right]^{-3/2}\left[\frac{-2v}{c^2}\right]\right]$$

$$\boxed{\frac{d\omega}{dv} = \frac{2\pi m_0 v}{h\left(1-\frac{v^2}{c^2}\right)^{3/2}}} \quad -(4)$$

diff eqn (3) w.r. to v

$$\frac{dk}{dv} = \frac{2\pi m_0}{h}\frac{d}{dv}\left[\frac{v}{\left(1-\frac{v^2}{c^2}\right)^{1/2}}\right]$$

$$\cancel{\frac{dk}{dv} = \frac{2\pi m_0}{h}\left[\frac{\left(1-\frac{v^2}{c^2}\right)^{1/2} - v\frac{1}{2}\left(1-\frac{v^2}{c^2}\right)^{-1/2}\left(\frac{-2v^2}{c^2}\right)}{(1-v^2/c^2)}\right]}$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h}\left[\frac{1\cdot\left(1-\frac{v^2}{c^2}\right)^{1/2} - v\cdot\frac{1}{2}\left(1-\frac{v^2}{c^2}\right)^{-1/2}\left(\frac{-2v}{c^2}\right)}{\left(1-\frac{v^2}{c^2}\right)}\right]$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h}\left[\frac{\left(1-\frac{v^2}{c^2}\right)^{1/2} + \frac{v^2/c^2}{\left(1-\frac{v^2}{c^2}\right)^{1/2}}}{1-\frac{v^2}{c^2}}\right]$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h}\left[\frac{1-\frac{v^2}{c^2}+\frac{v^2}{c^2}}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\right]$$

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$$\boxed{\frac{dk}{dv} = \frac{2\pi m_0}{h\left(1-\frac{v^2}{c^2}\right)^{3/2}}} \quad -(5)$$

From eqn (1), (4), (5) we get

$$v_g = \frac{2\pi m_0 v}{h\left(1-\frac{v^2}{c^2}\right)^{3/2}} \times \frac{h\left(1-\frac{v^2}{c^2}\right)^{3/2}}{2\pi m_0}$$

$$\boxed{v_g = v}$$ Hence proved

proof (ii) $v_p v_g = c^2$

we know that -

$$v_p = \frac{c^2}{v}$$

$$v_p = \frac{c^2}{v_g} \qquad \{v = v_g\}$$

$$\boxed{v_p v_g = c^2}$$ Hence proved

Physical significance of wave function - The wave function ψ as such has no physical significance but $|\psi|^2$ gives the probability density of finding the particle over the given region of space at any time.

for N - acceptable wave function it should be continuous, finite and single value everywhere.

$$\frac{\partial^2\psi}{\partial x^2} = \text{finite}, \qquad \frac{\partial\psi}{\partial x} = \text{continuous}$$

In the particle exist in the universe

$$\boxed{\int_{-\infty}^{\infty} |\psi|^2 \, dV = 1} \rightarrow \text{Normlization Condition}$$

This is known as Normlization condition.

→ If the particle does not exist –

$$\boxed{\int_{-\infty}^{\infty} |\psi|^2 dV = 0}$$

→ If the wave function is a complex function then $|\psi|^2 = \psi\psi^*$

where ψ^* = Complex Conjugate of ψ

ost
Schrondinger time independent and dependent wave eqn – [Time independent] –

The eqn of motion of a wave in differential form can be expressed as–

$$\boxed{\nabla^2\psi - \frac{1}{u^2}\frac{\partial^2\psi}{\partial t^2} = 0} \quad -(1)$$

The soln of eqn (1) is given by–

$$\boxed{\psi = \psi_0 e^{-i\omega t}} \quad -(2)$$

diff. eqn (2) twice with r. to t.

$$\boxed{\frac{\partial\psi}{\partial t} = -\psi_0 e^{-i\omega t} i\omega} \quad -(3)$$

$$\frac{\partial^2\psi}{\partial t^2} = -\psi_0(-i\omega) i\omega e^{-i\omega t}$$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2(\psi_0 e^{-i\omega t})$$

$$\boxed{\frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi} \quad -(4) \quad \text{put in (1) we get}$$

$$\boxed{\nabla^2\psi + \frac{\omega^2}{u^2}\psi = 0} \quad -(5)$$

$$\omega = 2\pi\nu = \frac{2\pi u}{\lambda} \qquad \{u \to \nu\lambda\}$$

$$\frac{\omega}{u} = \frac{2\pi}{\lambda} \qquad \left\{\lambda = \frac{h}{p}\right\}$$

$$\frac{\omega}{u} = \frac{2\pi p}{h}$$

$$\frac{\omega}{u} = \frac{p}{\hbar} \qquad \left\{\hbar = \frac{h}{2\pi}\right\}$$

H-cross

$$\frac{\omega^2}{u^2} = \frac{p^2}{\hbar^2}$$

Now,

$$E - V = \frac{p^2}{2m}$$

$$p^2 = 2m(E-V)$$

then

$$\boxed{\frac{\omega^2}{u^2} = \frac{2m}{\hbar^2}(E-V)} \to \text{put in eqn (5) we get}$$

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0} \quad -(6)$$

This is known as time independent wave eqn.

Time dependent wave eqn –

Imp Shrondinger Time dependent wave eqn – This eqn is optaing by elementing $E\psi$ from independent wave eqn. using eqn (3).

Now, eqn (3)

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0 e^{-i\omega t}$$

$$\frac{\partial \psi}{\partial t} = -i(2\pi\nu)\psi$$

$$\frac{\partial \psi}{\partial t} = -i\left(\frac{2\pi}{h}E\right)\psi$$

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar}\psi$$

$$\frac{\hbar}{-i}\frac{\partial \psi}{\partial t} = E\psi$$

$$\frac{i\hbar}{-i \times i}\frac{\partial \psi}{\partial t} = E\psi$$

$$\boxed{i\hbar\frac{\partial \psi}{\partial t} = E\psi} \quad -(7) \quad \text{put in eq}^n (6)$$

from eq^n (6)

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

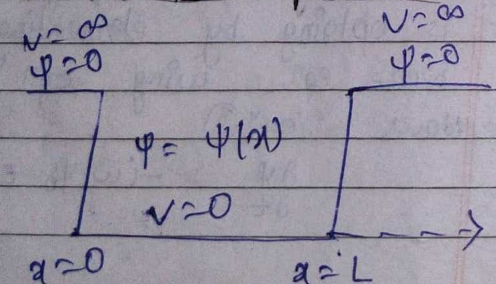
$$\nabla^2\psi + \frac{2m}{\hbar^2}\left(i\hbar\frac{\partial \psi}{\partial t} - V\psi\right) = 0$$

$$\boxed{i\hbar\frac{\partial \psi}{\partial t} = -\nabla^2\psi\frac{\hbar^2}{2m} + V\psi} \quad -(8)$$

$$\boxed{E\psi = H\psi} \quad \left\{H = \frac{-\hbar^2}{2m}\nabla^2 + V\right\}$$

Hamiltonian operator

particle in one dimensional potential well (Box) -



when a particle moving along x-axis. The well then its potential energy is described by -

$V = 0$	$0 \le x \le 1$
$V = \infty$	$L \le x \le 0$

The wave function for the particle is define as -

$\psi = \psi(x)$	$0 \le x \le L$
$\psi = 0$	$L \le x \le 0$

The motion of the particle inside the well is described by schrodinger time independent wave equation -

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

$$\boxed{\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0} \quad -(1)$$

The potential energy inside the well is zero.

$$V = 0$$

$$\boxed{\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}E\psi = 0} \quad -(2)$$

$$\text{Let } \left\{\frac{8\pi^2 m}{h^2}E = K^2\right\} \quad -(3)$$

$$\boxed{\frac{\partial^2\psi}{\partial x^2} + \psi K^2 = 0} \quad -(4)$$

The sol^n of eq^n (4) is given by

$$\psi = A\sin kx + b\cos kx \quad -(5)$$

Now, using boundary conditions we have-

$\boxed{x=0}$ then $\boxed{\psi=0}$

put eqⁿ (5)

$$0 = A\sin 0 + b\cos 0$$

$$B + 0 = 0$$

$$\boxed{B=0}$$

Again using Boundary condition we have-

$\boxed{x=L}$ then $\boxed{\psi=0}$ put eqⁿ (5)

$$0 = A\sin KL + b\sin KL$$

$$A\sin KL = 0$$

$A \neq 0$ then $\sin KL = 0$

$$\sin KL = \sin(\pm n\pi)$$

$$KL = \pm n\pi$$

$$\boxed{k = \frac{\pm n\pi}{L}} \quad -(6)$$

$$\boxed{k^2 = \frac{n^2\pi^2}{L^2}} \quad -(7)$$

but we have taken $k^2 = \frac{8\pi^2 mE}{h^2}$

$$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{L^2}$$

$$\boxed{E_n = \frac{n^2h^2}{8mL^2}} \quad -(8)$$

from here it is clear that the particle have ~~the~~ Discrete energy value which are known as Eigen value. } eigenvalue

then,

$$E_1 = \frac{h^2}{8mL^2}$$

$$E_2 = \frac{4h^2}{8mL^2} = 4E_1$$

$$E_3 = \frac{9h^2}{8mL^2} = 9E_1$$

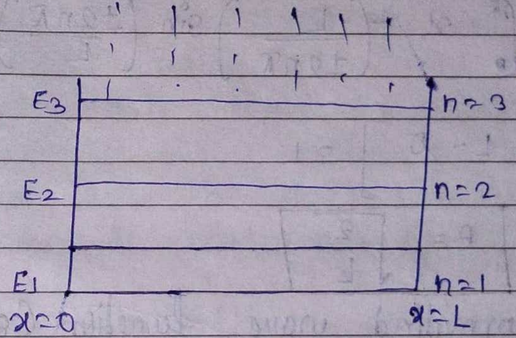


Fig- Eigen value diagram

The ~~eigen~~ function corresponding to each Eigen value is called eigen function.

from eqⁿ (5) we have-

$$\psi_n = A\sin kx$$ } Eigen function

eqⁿ(6) $\left\{ k = \frac{\pm n\pi}{L} \right\}$

$$\boxed{\psi_n = A\sin\left(\frac{\pm n\pi x}{L}\right)} \quad -(9)$$

To Normlize the wave function we used normalized condition.

then $\int_0^L |\psi_n|^2 dx = 1$

$$\int_0^L A^2 \sin^2\left(\frac{\pm n\pi x}{L}\right) dx = 1$$

$$\left\{ \begin{array}{l} \cos 2\theta = 1 - 2\sin^2\theta \\ \sin^2\theta = \frac{1-\cos 2\theta}{2} \end{array} \right\}$$

$$A^2 \int_0^L \left[1 - \cos\left(\frac{\pm 2n\pi x}{L}\right)\right] dx = 1$$

$$\frac{A^2}{2} \left[x - \left(\frac{L}{\pm 2n\pi}\right) \sin\left(\frac{\pm 2n\pi}{L}\right) \right]_0^L = 1$$

$$\frac{A^2}{2}\left[L - 0 \right] = 1$$

$$\boxed{A = \sqrt{\frac{2}{L}}}$$

then normalized wave function (Eigen function)

$$\boxed{\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{\pm n\pi x}{L}\right)} \quad -10$$

$$\boxed{\psi_2 = \sqrt{\frac{2}{L}} \sin\left(\frac{\pm 2\pi x}{L}\right)}$$

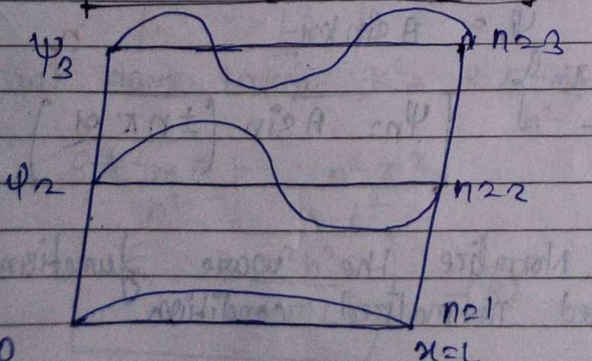
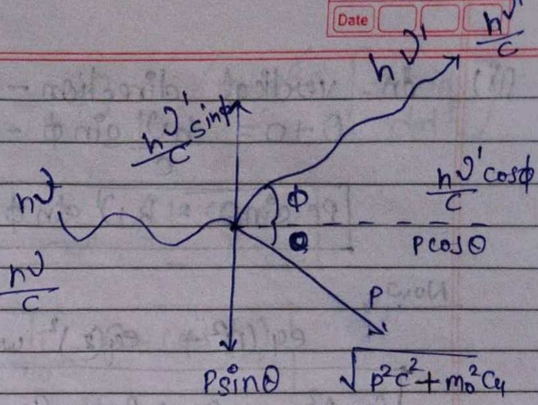


Fig- Eigen function Diagram

✱ Compton effect-



when x-Ray incident on a crystal when then there are two types of radition accurs in scatterid reagion.

① which have larger wavelength is called modify radition and another Radition which have same wavelength as the wavelength of incident photon is called unmodifieid sti radition.

The compton shift depends only upon the angle of scattering.
i.e. It does not depend upon the wavelength of incident photon.

According to conservation Law of momentum.

(i) In horizontal direction -

$$\left\{ E = \frac{h\nu}{c} = \frac{mc^2}{c} = mc \right\}$$

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + P\cos\theta$$

$$\boxed{Pc\cos\theta = h\nu - h\nu'\cos\phi} \quad ①$$

(ii) In vertical direction –

$$0+0 = \frac{h\nu'}{c}\sin\phi - P\sin\theta$$

$$\boxed{Pc\sin\theta = h\nu'\sin\phi} \quad -(2)$$

Now,

eq $(1)^2 +$ eq $(2)^2$ we get –

$$\boxed{(Pc)^2 = (h\nu)^2 + (h\nu')^2 - 2h\nu h\nu'\cos\phi} \quad -(3)$$

Acc. to conservation Law of energy –

$$h\nu + m_0c^2 = h\nu' + \sqrt{(Pc)^2 + m_0^2c^4}$$

$$[h\nu - h\nu' + m_0c^2]^2 = (Pc)^2 + m_0^2c^4$$

$$(h\nu)^2 + (h\nu')^2 - 2h\nu h\nu' + m_0^2c^4 + 2m_0c^2(h\nu - h\nu') = P^2c^2 + m_0^2c^4$$

$$\boxed{P^2c^2 = (h\nu)^2 + (h\nu')^2 - 2h\nu h\nu' + 2m_0c^2(h\nu - h\nu')} \quad -(4)$$

∴ from eq^n (3) and (4) we get

$$(h\nu)^2 + (h\nu')^2 - 2h\nu h\nu'\cos\phi = (h\nu)^2 + (h\nu')^2 - 2h\nu h\nu' + 2m_0c^2(h\nu - h\nu')$$

$$h\nu h\nu'[1-\cos\phi] = m_0c^2[h\nu - h\nu']$$

$$\frac{hc\,hc}{\lambda\lambda'}[1-\cos\phi] = m_0c^2\left[\frac{c}{\lambda} - \frac{c}{\lambda'}\right] \quad \left\{\nu = \frac{c}{\lambda}\right\}$$

$$\frac{hc\,hc}{\lambda\lambda'}[1-\cos\phi] = h\,m_0c^2c\left[\frac{\lambda'-\lambda}{\lambda\lambda'}\right]$$

$$\boxed{\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\phi)}$$

$$\boxed{\Delta\lambda = \frac{h}{m_0c}(1-\cos\phi)}$$

↳ compton shift.

Case-I when $\phi = 0$ then $\Delta\lambda = 0$ i.e. unmodified radition is obtain in the direction of incident photon

Case-II when $\phi = \frac{\pi}{2}$ then $\Delta\lambda = \frac{h}{m_0c} = \lambda_c$

(Compton wavelength)

$$\lambda_c = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}$$

$$\lambda_c = 0.0243\ \mathring{A}$$

The compton wavelength is obtain in the direction of Normal to the incident photon.

Case-III when $\phi = \pi$ then $\Delta\lambda = \Delta\lambda_{max} = \frac{2h}{m_0c}$

Que. Why Compton effect is not observed in visible Light Region? $\Delta\lambda_{max} = 0.0486\ \mathring{A}$

most Imp

thus compton effect is not observed in visible light.

because the change in wave length is negligible in comparison to the wave length of visible light

Que. Calculate the De-Brogli wavelength if the particle is n-electron and potential difference $V = 50$ volt e?

Ans.

$$\lambda = \frac{h}{\sqrt{2mqV}} = \frac{12.28}{\sqrt{V}} \ A^0$$

$$\lambda = \frac{12.28}{\sqrt{50}} \ A^0$$

$$\boxed{\lambda = 1.7366 \ A^0}$$

Que. Calculate D. broglie wavelength of a 15KV electron.

Sol^n -

$$\lambda = \frac{h}{\sqrt{2mE}} \qquad \left\{ \begin{aligned} E &= 15 keV \\ &= 15 \times 10^3 \times 1.6 \times 10^{-19} \ J \end{aligned} \right.$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 15 \times 10^3 \times 1.6 \times 10^{-19}}}$$

$$\lambda = 0.1 \ A^0$$

Que. Calculate the velocity and K.E. of a 1 Nutron having D.b.w 1 A^0

Sol^n -
$V = ?$
$E = ?$

$$\lambda = \frac{h}{mv}$$

$$v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1 \times 10^{-10}}$$

$$\boxed{v = 3.96 \times 10^3 \ m/sec}$$

$$\lambda_n = \frac{0.286}{\sqrt{E_{eV}}} \ A$$

$$\sqrt{E_{eV}} = \frac{0.286}{\lambda} \times 10^{-10}$$

$$\sqrt{E_{eV}} = \frac{0.286}{1 \times 10^{-10}} \times 10^{-10}$$

$$\boxed{E_{eV} = 0.082 \ eV}$$

Imp Que. Calculate D.b.w. of an α particle exalated through a potencial difference of 200 V.

Sol^n -
$m_\alpha = 4m_n = 4 \times 1.67 \times 10^{-27}$ kg
$q_\alpha = 2 \times e = 2 \times 1.6 \times 10^{-19}$ C

$$\lambda = \frac{h}{\sqrt{2 m_\alpha q_\alpha V}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 4 \times 1.67 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times 200}}$$

$\lambda = 0.00716 \ A^0$ OR 716×10^{-13} m

Que. Calculate the D.B.W. of a nutron having K.E. of 1eV

Sol^n -

$$\lambda_n = \frac{0.286}{\sqrt{E_{eV}}} \ A^0$$

$$\lambda_n = \frac{0.286}{\sqrt{1}}$$

$$\boxed{\lambda_n = 0.286 \ A^0}$$ Ans.

Que. Can a photon and an electron of the same momentum have the same wavelength compair their wavelength in the the n have a same energy

Sol

(i) Given $P_{ph} = P_e$

$$d_{ph} = \frac{h}{P_{ph}} \quad -(1)$$

$$d_e = \frac{h}{P_e} \quad -(2)$$

eq(1) / (2)

$$\frac{d_{ph}}{d_e} = \frac{h}{d_{ph}} \times \frac{P_e}{h} \quad \{P_{ph} = P_e\}$$

$$\frac{d_{ph}}{d_e} = \frac{P_e}{d_{ph}} = 1$$

$$\boxed{d_{ph} = d_e}$$

(ii) Given $E_{ph} = E_e = E$ (say)

$$d_e = \frac{h}{\sqrt{2mE_e}} \quad -(1) \qquad \left\{ E = \frac{hc}{d} \right.$$

$$d_{ph} = \frac{hc}{E_{ph}} \quad -(2) \qquad \left. d = \frac{hc}{E} \right\}$$

eq (1) / eq (2)

$$\frac{d_e}{d_{ph}} = \frac{h}{\sqrt{2mE_e}} \times \frac{E}{hc} \quad \{E_{ph} = E_e = E\}$$

$\{E = mc^2\}$

$$\boxed{\frac{d_e}{d_{ph}} = \sqrt{\frac{E}{2mc^2}}} = \sqrt{\frac{E}{2E}} = \frac{1}{\sqrt{2}} \text{ Ans.}$$

Que. Calculate D.B.W associated with Nitrogen 30 atmospheric pressure and temp. 27°c and mass 4.65×10^{-26} kg.

Sol –

$T = 27°C = 300K$

$M_n = 4.65 \times 10^{-26}$ kg

$$d = \frac{h}{\sqrt{3mKT}} \qquad \{K = 1.38 \times 10^{-23}\}$$

$$d = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 4.65 \times 10^{-26} \times 1.38 \times 10^{-23} \times 300}}$$

$$\boxed{d = 0.2754 \text{ Å}} \text{ Ans.}$$

Imp

Que. N electron is bound in 1 dimensional potential well which has width 2.5×10^{-10}m assume the hight of the well to be ∞. Calculate the lowest two permitted energy values of e^-.

Ans.

$$E_n = \frac{n^2 h^2}{8mL^2}$$

$$E_n = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} (2.5 \times 10^{-10})} \times n^2$$

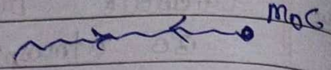
$$E_n = 9.66 \times 10^{-19}\, n^2 \text{ Joul}$$

Now

$E_1 = 9.66 \times 10^{-19}$ Joul

$E_2 = 19.32 \times 10^{-19}$ Joul

Que. A photon recoils back after striking an electron at west. what is the change in wavelength of the photon. $\phi = \pi$



Ans.

$$\Delta\lambda = \frac{h}{m_0c}(1-\cos\phi)$$

$$\Delta\lambda = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-(-1))$$

$$\boxed{\Delta\lambda = 0.0486\ \mathring{A}}$$ Ans.

Q. In a compton experiment exlarate of wavelength $0.015\ \mathring{A}$ is scratted at 60° find the wavelength scratted wavelength.

Soln– $\phi = 60^\circ$

$\lambda' = ?$

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0C}(1-\cos\phi)$$

$$\Delta\lambda = \lambda' - 0.015 = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos60)$$

$$= \lambda' - 0.015 = 0.0243\left(1-\frac{1}{2}\right)$$

$$\lambda' = \frac{0.0243}{2} + 0.015$$

$$\boxed{\lambda' = 0.027\ \mathring{A}}$$

Que. An x-Ray photon is found to have its wavelength doubled on been scratted to 90°. find the wavelength and energy of incident photon.

Ans. $\phi = 90^\circ$

$\lambda = ?$

$E = ?$

$$\lambda' - \lambda = \frac{h}{m_0C}(1-\cos90)$$

$$2\lambda - \lambda = 0.0243\,(1-0) \quad \{\because \lambda' = 2\lambda\}$$

$$\boxed{\lambda = 0.0243\ \mathring{A}}$$

(ii) $$E = h\nu = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{0.0243\times10^{-10}}$$

$$\boxed{E = 8.106\times10^{-14}\ \text{Joul}}$$

Questions

{Mark 2}

1. Define De brogli Hypothesis?
2. Define phase wave and group velocity?
3. Physical significance of wave function?
4. Define Eigen values and Eigen function?
5. what is modify and demodify radition?
6. why compton effect is not observed in visible Region?

{Mark-7}

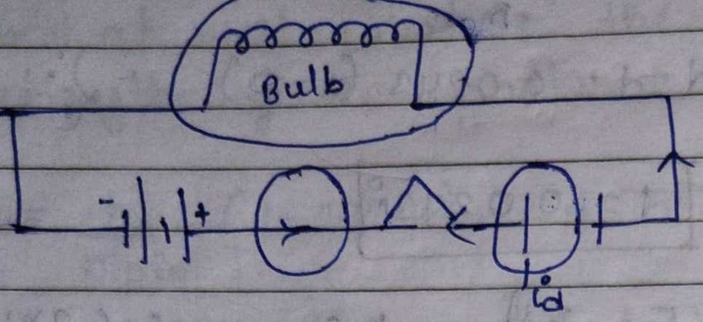
7. Derive different expression of De Brogli wavelength?
8. (i) $v_g = v$ (ii) $v_p v_g = c^2$
9. Shrondingar time dependent and independent?
10. what is Compton effect?

UNIT- 2

Electromagnatics

(2) mark

Displacement current-



- Maxwell introduce that the magnatic field is produce by change in eletric field.
- The current flowing due to the change in electric field b/w the plate of capacitor is called displacement current.

$$i_d = \varepsilon_0 A \frac{\partial \vec{E}}{\partial t}$$

$$i_d = A \frac{\partial \vec{D}}{\partial t}$$

{ where $\vec{D} = \varepsilon_0 \vec{E}$ Electric displacement vector }

The magnatic field produce by displacement current is similar to the magnatic field produce by conventional current.

It produce as long as electric field is changing.

Gauss divergence theorem-

According to Gauss divergence theorm the surface integral may be change to volume integral by taking the divergence of given vector.

i.e. $$\boxed{\oint_S \vec{A}\cdot \vec{ds} = \int_V \nabla\cdot \vec{A}\, dV}$$

Stoke's theorm-

According to stoke's theorm the line Integral may be change to surface integral by taking the curl of the given vector.

i.e.

$$\boxed{\oint_C \vec{A}\cdot \vec{dl} = \oint_S \nabla\times \vec{A}\cdot \vec{ds}}$$

Equation of Continuity - The current is define

Imp as the Ratio of change to the time take.

i.e. $$i = \frac{q}{t}$$

current is also define as-

$$i = \oint_S \vec{J}\cdot \vec{ds} \quad -(1)$$

$\because$ J = current density

For a small volume element the current is also define as weight Rate of ~~negative~~ decrease of ~~two~~ -tive charge with in that element.

i.e. $$i = \frac{-dQ}{dt} \quad (2)$$

The volume change density is given by

$$\rho = \frac{Q}{V}$$

the $Q = \int_V \rho\, dv$

the $$i = -\frac{\partial}{\partial t}\int_V \rho\, dv \quad (3)$$

From eq^n ① and ③

$$\oint_S \vec{J}\cdot\vec{ds} = -\frac{\partial}{\partial t}\int_V \rho\, dv$$

Now the surface integral may be change to volume integral using gauss divergence theorm-

then

$$\int_V \nabla\cdot\vec{J}\cdot dv = -\frac{\partial}{\partial t}\int_V \rho\, dv$$

then

$$\int_V \left(\nabla\cdot\vec{J} + \frac{\partial \rho}{\partial t}\right) dv = 0$$

$$\left\{\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0\right\}$$

then

$$\boxed{\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}} \quad -④$$

$$\boxed{\text{div}\,\vec{J} = -\frac{\partial\rho}{\partial t}}$$

Eq^n ④ is known as eq^n of continuity.

Imp The physical significance of en^n of Continuity is that " The charge diverging from a small volume element is the current passing through the element."

physical significance of continuity eq^n-

Imp

The charge diverging from a small element is equal to the rate of decrease charge with that element.

Maxwell's ~~Electromagnatic wave~~ eq^n – All the fundamental Law of electricity and magnitim are formulate in terms of Four equation which are known as maxwell Em wave eq^n.

maxwell also modified Amp. circital law for static as well as dynamic charges.

Ⓐ Maxwell Eq^n in Free Space –

$\rho = 0$ $\qquad$ $\vec{J} = 0$

(i) $\nabla \cdot \vec{E} = 0$
OR $\nabla \cdot \vec{D} = 0$ $\;\Big] \rightarrow$ ① $\quad \{ \because \vec{D} = \varepsilon_0 \vec{E} \}$

[Gauss Law of Electrostatic]

(ii) $\nabla \cdot \vec{B} = 0$ – ② $\quad \{ \because \vec{B} = \mu_0 \vec{H} \}$

{Gauss Law of magnostatic}

(iii) $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$ – ③ [Farade Law of Electro magnatic Induction]

(iv) $\nabla \times \vec{H} = \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$ – ④

[Modify form of Amp. Circital Law]

Ⓑ Maxwell Eq^n in differential form –

(i) $\nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0}$
OR $\nabla \cdot \vec{D} = \rho$ $\;\Big] \rightarrow$ ①

(ii) $\nabla \cdot \vec{B} = 0$ – ②

(iii) $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$ – ③

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Date

Maxwell's Electromagnatic wave eqⁿ- All the fundamental Law of electricity and magnitim are formulate in terms of Four equation which are known as maxwell Em wave eqⁿ.

maxwell also modified Amp. circital law for static as well as dynamic charges.

(A) Maxwell Eqⁿ in Free Space -

$\rho = 0$ $\quad \vec{J} = 0$

(i) $\nabla \cdot \vec{E} = 0$
OR $\nabla \cdot \vec{D} = 0$ → (1) $\{\because \vec{D} = \varepsilon_0 \vec{E}\}$
[Gauss Law of Electrostatic]

(ii) $\nabla \cdot \vec{B} = 0$ - (2) $\{\because \vec{B} = \mu_0 \vec{H}\}$

{Gauss Law of magnostatic}

(iii) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ - (3) [Farade Law of Electro magnatic Induction]

(iv) $\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ - (4)
[Modify form of Amp. Circital law]

(B) maxwell Eqⁿ in differential form-

(i) $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$
OR $\nabla \cdot \vec{D} = \rho$ → (1)

(ii) $\nabla \cdot \vec{B} = 0$ - (2)

(iii) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ - (3)

(iv) $\nabla \times \vec{H} = \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ — (4) Imp

(c) maxwell Eqn in Integral form-

(i) $\oint_S \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_0} \int_V \rho \, dv$ —(1)

(ii) $\oint_S \vec{B} \cdot \vec{ds} = 0$ —(2)

(iii) $\oint_C \vec{E} \, \vec{dl} = -\int_S \frac{\partial \vec{B}}{\partial t} \vec{ds}$ —(3)

(iv) $\oint_C \vec{H} \cdot \vec{dl} = \oint_S \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \vec{ds}$ —(4)

Derivation of maxwell's in differental form-

(i) First eqn →

$$\boxed{\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}}$$

According to gauss Law of Electrostatic

$$\oint_S \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_0} \int_V \rho \, dv$$

using Gauss divergence theorm-

$$\int_V \nabla \cdot \vec{E} \cdot dv = \frac{1}{\varepsilon_0} \int_V \rho \, dv$$

$$\int_V \left(\nabla \cdot \vec{E} - \frac{1}{\varepsilon_0} \rho \right) dv = 0$$

$$\nabla \cdot \vec{E} - \frac{\rho}{\varepsilon_0} = 0$$

$$\boxed{\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}}$$ proved

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(ii) Second eqⁿ → $\boxed{\nabla \cdot \vec{B} = 0}$

According to Gauss Law of ~~Electromagnatic~~ - magnostatic -

$$\oint_S \vec{B} \cdot \vec{ds} = 0$$

Using Gauss divergance theorm-

$$\int_V \nabla \cdot \vec{B} \, dv = 0$$

$$\boxed{\nabla \cdot \vec{B} = 0}$$

proved

(iii) Third eqⁿ -

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

The magnatic flux in term of magnatic flux density is define as

$$\phi = \oint_S \vec{B} \cdot \vec{ds} \quad -(1)$$

According to Farade Law of Electromagnetic Induction-

$$e = -\frac{\partial \phi}{\partial t}$$

$$\boxed{e = -\oint_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}} \quad -(2)$$

Let $\vec{E}$ is the Electric field of line Element dl for a closed loop the EmF is define as the workdone to carry a unit charge or the closed loop that is-

$$\boxed{e = \oint_C \vec{E} \cdot \vec{dl}} \quad -③$$

From eq^n ② & ③

$$\oint_C \vec{E} \cdot \vec{dl} = -\oint_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

using stoke's theorm-

$$\oint_S \nabla \times \vec{E} \cdot \vec{ds} = -\oint_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

proved

most Imp

(iv) Fourth equation -

Or Displacement current

$$\nabla \times \vec{H} = \vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Amperar circital law is define as-

$$\oint_C \vec{H} \cdot \vec{dl} = i \quad -①$$

$$\oint_C \vec{H} \cdot \vec{dl} = \oint_S \vec{J} \cdot \vec{ds} \quad -②$$

by using stoke's theorm-

$$\oint_S \nabla \times \vec{H} \cdot \vec{ds} = \oint_S \vec{J} \cdot \vec{ds}$$

$$\boxed{\nabla \times \vec{H} = \vec{J}} \quad - (3)$$

Taking divergence both sides.

$$\nabla \cdot \nabla \times \vec{H} = \nabla \cdot \vec{J}$$

$$\boxed{\nabla \cdot \vec{J} = 0} \quad - (4) \qquad \{\nabla \cdot \nabla \times \vec{H} = 0\}$$

but, the eqn According to eqn of continuity-

$$\boxed{\nabla \cdot \vec{J} = \frac{-\partial \rho}{\partial t}} \quad - (5)$$

If eqn 4th and 5th is valid then ρ = const.
i.e. Amp. circital Law is valid only for static charge. maxwell modified it for static as well as dynamic charges by adding a term -
$\vec{J}_d$ on R.H.S of eqn (3)

$$\boxed{\nabla \times \vec{H} = \vec{J} + \vec{J}_d} \quad - (6)$$

Taking divergamce both sides

$$\nabla \cdot \nabla \times \vec{H} = \nabla \cdot \vec{J} + \nabla \cdot \vec{J}_d$$

$$\nabla \cdot \vec{J} + \nabla \cdot \vec{J}_d = 0$$

$$\boxed{\nabla \cdot \vec{J}_d = - \nabla \cdot \vec{J}}$$

put value in eq^n (5)

$$\nabla \cdot \vec{J}_d = \frac{\partial \rho}{\partial t} \quad \left\{ \begin{array}{l} \text{From maxwell} \\ \text{Ist eq}^n\ \nabla \cdot \vec{D} = \rho \end{array} \right\}$$

$$\nabla \cdot \vec{J}_d = \frac{\partial}{\partial t}(\nabla \cdot \vec{D})$$

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} \quad \{ \vec{D} = \varepsilon_0 \vec{E} \}$$

$$\vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

put in eq^n (6)

$$\boxed{\nabla \times \vec{H} = \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

Hence proved

Derivation of maxwell's eq^n in integral form-

(i) First eq^n-

$$\boxed{\oint_S \vec{E} \cdot \vec{dS} = \frac{1}{\varepsilon_0} \int_V \rho \, dV}$$

maxwell first eq^n Differential form can be Expressed as-

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

Taking volume integral both side-

$$\int_V \nabla \cdot \vec{E} \, dV = \frac{1}{\varepsilon_0} \int \rho \, dV$$

using Gauss divergance theorm-

$$\boxed{\oint_S \vec{E}\cdot\vec{ds} = \frac{1}{\varepsilon_0}\int_V \rho\, dV}$$

proved

(ii) Second Eq^n-

$$\oint_S \vec{B}\cdot\vec{ds} = 0$$

maxwell second eq^n in differtial form -

$$\nabla\cdot\vec{B} = 0$$

Taking volume Integral

$$\int_V \nabla\cdot\vec{B}\cdot dV = 0$$

Using Gauss divergance theorm-

$$\boxed{\oint_S \vec{B}\cdot\vec{ds} = 0}$$

(iii) Third Eq^n-

$$\boxed{\oint_C \vec{E}\,\vec{dl} = -\int \frac{\partial\vec{B}}{\partial t}\,\vec{ds}}$$

maxwell Third eq^n in differential form is defined as-

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

Taking surface integral both sides

$$\oint_S \nabla\cdot\vec{E}\cdot\vec{ds} = \oint_S -\frac{\partial\vec{B}}{\partial t}\cdot\vec{ds}$$

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by using stoke's theorm-

$$\boxed{\oint_C \vec{E}\cdot\vec{dl} = -\oint_S \frac{\partial \vec{B}}{\partial t}\cdot ds}$$

proved

Fourth eq^n-

$$\oint_C \vec{H}\cdot\vec{dl} = \oint_S \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)\vec{ds}$$

maxwell Foureth eq^n in differential form is defined as-

$$\nabla \times \vec{H} = \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Taking surface integral-

$$\oint_S \nabla\times\vec{H}\cdot\vec{ds} = \oint_S \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)\cdot\vec{ds}$$

by stoke's theorm-

$$\boxed{\oint_C \vec{H}\cdot\vec{dl} = \oint_S \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)\cdot\vec{ds}}$$

Proved

Physical significance of maxwell's eq^n-

First eq^n- Is Introduce that total Electric flux and closed by Hypothetical gaussin surface is $\frac{1}{\varepsilon_0}$ times the total charge and closed with in that surface.

Thus, It represents gauss Law of electrostatics.

② Second eq^n -
It Introduce that magnatic monopoll doesn't exist. So It represent Gauss Law of magnatostatics.

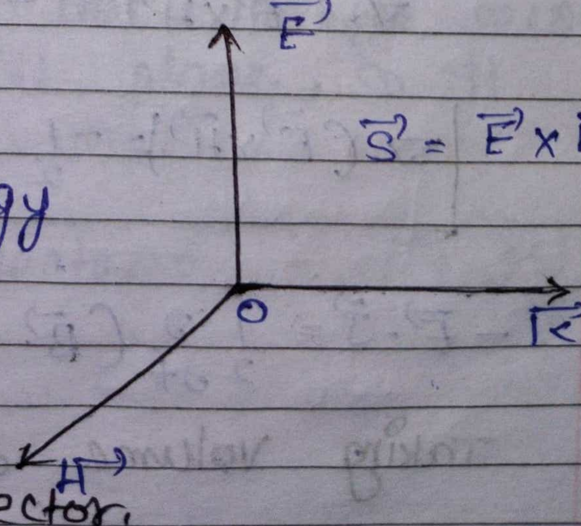
③ Third eq^n -
It Introduce that the rate of change of magnatic flux produced an induced emf. Hence, It represent Farade Law of Electromagnatic ~~Radition~~ Induction.

④ Fourth eq^n - It stated that the magnatic field is produced by conventional current as well as change in electric field. It represents modified form of Ampear's circital Law.

Very Imp

Poynting vector -

Numerical - 7
Derivation - 7
Short - 2



$\vec{S} = \vec{E} \times \vec{H}$

The amount of Energy crossing per unit area per second perpendicular to E-H plane is known as poynting vector.

i.e. $\vec{S} = \vec{E} \times \vec{H}$

$\vec{S} = EH \sin\theta \, \hat{n}$

$$\boxed{|\vec{S}| = EH \quad \text{Joul/sec-m}^2}$$

poynting theorm-

maxwell third and 4rth eq^n in differential form can be expressed as-

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (1)$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \vec{J} \quad (2)$$

Taking by dot product by $\vec{H}$ and $\vec{E}$ with eq^n (1) & (2) respectively-

$$\vec{H} \cdot \nabla \times \vec{E} = -\mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} \quad (3)$$

$$\vec{E} \cdot \nabla \times \vec{H} = \varepsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} + \vec{E} \cdot \vec{J} \quad (4)$$

we know that-

$$\nabla \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot \nabla \times \vec{E} - \vec{E} \cdot \nabla \times \vec{H} \quad (5)$$

$$\nabla \cdot (\vec{E} \times \vec{H}) = -\mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} - \varepsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} - \vec{E} \cdot \vec{J}$$

$$\boxed{\nabla \cdot (\vec{E} \times \vec{H}) = -\frac{1}{2} \frac{\partial}{\partial t} (\mu \vec{H} \cdot \vec{H} + \varepsilon \vec{E} \cdot \vec{E}) - \vec{E} \cdot \vec{J}} \quad (6)$$

$$-\vec{E} \cdot \vec{J} = \frac{1}{2} \frac{\partial}{\partial t} (\vec{B} \cdot \vec{H} + \vec{D} \cdot \vec{E}) + \nabla \cdot (\vec{E} \times \vec{H})$$

Taking volume Integral on both sides-

$$-\int_V \vec{E}\cdot\vec{J}\,dv = \frac{1}{2}\frac{\partial}{\partial t}\int_V (\vec{B}\cdot\vec{H} + \vec{D}\cdot\vec{E})\,dv + \int_V \nabla\cdot(\vec{E}\times\vec{H})\,dv$$ → Gauss div. theorm

$$\boxed{\underbrace{-\int_V \vec{E}\cdot\vec{J}\,dv}_{\text{Ist term}} = \underbrace{\frac{1}{2}\frac{\partial}{\partial t}\int_V (\vec{B}\cdot\vec{H} + \vec{D}\cdot\vec{E})\,dv}_{\text{second term}} + \underbrace{\oint_S (\vec{E}\times\vec{H})\cdot d\vec{s}}_{\text{third term}} \quad (7)}$$

This is known as poynthing theorm or Conservation Low of energy in electro magnatice wave.

The physical significance of pointing theorm-

(I) Ist term- It represent the transmission of energy due to the motion of charge.

(II) IInd term- It represent the sum of magnatic and electric potencial energy's.

III IIIrd term- It represent the amount of energy crossing per unit areaper second perpendicular to EH plane, so It represents pointing vector.

$$\boxed{\text{poynting vector} = \vec{E}\times\vec{H} = \vec{S}}$$

V. Imp

Propgation of E M wave in free space-

maxwell Electromagnatic wave Eq^n in free space can be discribed as-

$$\nabla \cdot \vec{E} = 0 \quad -(1)$$

$$\nabla \cdot \vec{H} = 0 \quad -(2)$$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad (3)$$

$$\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad -(4)$$

Taking curl both sides in eq^n (3)

$$\nabla \times \nabla \times \vec{E} = -\mu_0 \frac{\partial}{\partial t} \nabla \times \vec{H}$$

$$\boxed{\nabla \times \nabla \times \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \vec{E}} \quad \{\text{using } eq^n \text{ (4)}\} \quad -(5)$$

We know that -

$$\boxed{\vec{A} \times \vec{B} \times \vec{C} = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})} \quad -(6)$$

by comparing eq^n (5) and (6)

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - (\nabla \cdot \nabla)\vec{E}$$

$$\boxed{\nabla \times \nabla \times \vec{E} = 0 - \nabla^2 \vec{E}} \quad \{eq^n \text{ (1) using}\}$$

put eq^n (5)

$$-\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \quad -(7)$$

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similarly on taking the curl on eq^n (4) both side we have-

$$\boxed{\nabla^2 \vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}} \quad -(8)$$

we obtain two similar electromagnatic wave equation $\vec{E}$ and another $\vec{H}$ for thus the general equation of EM wave is given by-

$$\boxed{\nabla^2 \phi = \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2}} \quad -(9)$$

Compare eq^n 7, 8, (9)

$$\frac{1}{v^2} = \mu \varepsilon_0$$

$$\boxed{v = \frac{1}{\sqrt{\mu \varepsilon_0}}}$$

Now,

$$\mu_0 = 4\pi \times 10^{-7}$$

$$\varepsilon_0 = \frac{1}{4\pi \times 9 \times 10^9}$$

$$v = \cancel{\frac{1}{\sqrt{4\pi \times 10^{-7} \times 4\pi \times 9 \times 10^9}}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{1}{4\pi \times 9 \times 10^9}}}$$

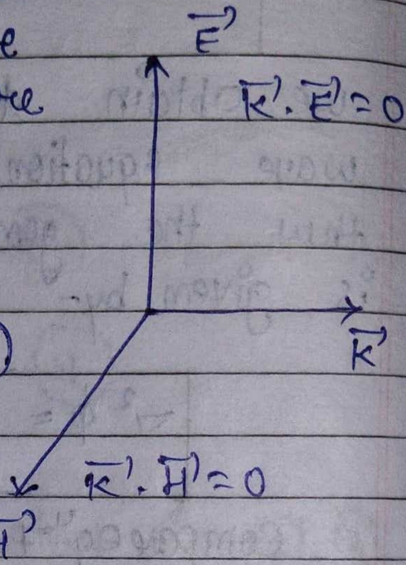
$$v = \frac{1}{(10^{-8}/3)} \quad \therefore \quad 3 \times 10^8 = c \ m/s$$

$$\boxed{v = c = 3 \times 10^8 \ m/s}$$

Thus, Electromagnetic wave travel with the speed of light in free space (vacuum).

Transverse nature of EM wave-

The Electromagnetic wave eq^n for $\vec{E}$ and $\vec{H}$ are given by -



$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad -(1)$$

$$\nabla^2 \vec{H} - \frac{1}{c^2}\frac{\partial^2 \vec{H}}{\partial t^2} = 0 \quad -(2)$$

The sol^n of eq^n (1) & and eq^n (2) are given by

$$\vec{E} = \vec{E_0}\, e^{i(\vec{K}.\vec{r} - \omega t)} \quad -(3)$$

$$\vec{H} = \vec{H_0}\, e^{i(\vec{K}.\vec{r} - \omega t)} \quad -(4)$$

The maxwell I^st and II^nd eq^n in Free space are defined as-

$$\nabla . \vec{E} = 0 \quad -(5)$$
$$\nabla . \vec{H} = 0 \quad -(6)$$

put value in eq^n (5) of $\vec{E}$

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$$\nabla \cdot \vec{E_0}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)} = 0$$

$$i\vec{k}\cdot \underbrace{\vec{E_0}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}}_{\vec{E}} = 0$$

$$i\,\vec{k}\cdot\vec{E} = 0$$

$$\boxed{\vec{k}\cdot\vec{E} = 0} \quad -(7)$$

$\vec{k} \perp \vec{E}$

From eq^n 4^{rth} and 6th

$$\nabla \cdot \vec{H_0}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)} = 0$$

$$i\cdot\vec{k}\cdot\vec{H} = 0$$

$$\boxed{\vec{k}\cdot\vec{H} = 0} \quad -(8)$$

$\vec{k} \perp \vec{H}$

From eq^n (7) and eq^n (8) It is clear that k is perpendicular to both $\vec{E}$ and $\vec{H}$ thus Electromagnatic wave is transverse in nature.

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{E_0} = E_{0x}\hat{i} + E_{0y}\hat{j} + E_{0z}\hat{k}$$

$$\vec{k} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{k}\cdot\vec{r} = xk_x + yk_y + zk_z$$

$$\vec{k}\cdot\vec{E_0} = k_x E_{0x} + k_y E_{0y} + k_z E_{0z}$$

Now,

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(E_{0x}\hat{i} + E_{0y}\hat{j} + E_{0z}\hat{k}\right) e^{i(xk_x + yk_y + zk_y)} = 0$$

$$E_{0x}\frac{\partial}{\partial x} e^{i(xk_x + yk_y + zk_z - \omega t)} + E_{0y}\frac{\partial}{\partial y} e^{i(xk_x + yk_y + zk_z - \omega t)} + E_{0z}\frac{\partial}{\partial z} e^{i(xk_x + yk_y + zk_z)} = 0$$

$$(iE_{0x}k_x + iE_{0y}k_y + iE_{0z}k_z) e^{i(\vec{k}\cdot\vec{r} - \omega t)} = 0$$

$$i(k_x E_{0x} + k_y E_{0y} + k_z E_{0z}] e^{i(\vec{k}\cdot\vec{r} - \omega t)} = 0$$

$$i\vec{k}\cdot\vec{E_0}\, e^{i(\vec{k}\cdot\vec{r} - \omega t)} = 0$$

$$i\vec{k}\cdot\vec{E} = 0$$

$$\boxed{\vec{k}\cdot\vec{E} = 0} \quad - (7)$$

$$\boxed{\vec{k} \perp \vec{E}}$$

Similarly-

$$\nabla\cdot\vec{H_0}\, e^{i(k\cdot r - \omega t)} = 0$$

$$i\vec{k}\cdot\vec{H} = 0$$

$$\boxed{\vec{k}\cdot\vec{H} = 0} \quad - (8)$$

$$\boxed{\vec{k} \perp \vec{H}}$$

The maxwell III^{rd} and IV^{th} eq^n can be expressed as -

$$\boxed{\nabla\times\vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}} \quad -(9)$$

$$\boxed{\nabla\times\vec{H} = \epsilon_0 \frac{\partial}{\partial t}\vec{E}} \quad -(10)$$

From eq^n (3), (4) & (9) we have-

$$\nabla\times\vec{E_0}\, e^{i(\vec{k}\cdot\vec{x}-\omega t)} = -\mu_0 \frac{\partial}{\partial t} \vec{H_0}\, e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$i\vec{k}\times\vec{E} = i\omega\mu_0\vec{H}$$

$$\boxed{\vec{k}\times\vec{E} = \omega\mu_0\vec{H}} \quad -(11)$$

From eq^n (3), (4) & (10) -

$$\nabla\times\vec{H_0}\, e^{i(\vec{k}\cdot\vec{x}-\omega t)} = \epsilon_0 \frac{\partial}{\partial t} E_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$i\vec{k}\times\vec{H} = -i\omega\epsilon_0\vec{E}$$

$$\boxed{\vec{k}\times\vec{H} = -\vec{E}\omega\epsilon_0} \quad -(12)$$

From eq^n (7), (8), (11) & (12) - It is clear that E, H and K are mutully perpendicular.

From eq^n (11) we have-

$$K\times\vec{E} = \mu_0\omega\vec{H}$$
$$K\hat{n}\times\vec{E} = \mu_0\omega\vec{H}$$
$$K|\hat{n}\times\vec{E}| = \mu_0\omega|\vec{H}|$$
$$K|\vec{E}| = \mu_0\omega|\vec{H}|$$
$$\frac{|\vec{E}|}{|\vec{H}|} = \frac{\mu_0\omega}{k} = \mu_0 c \quad \left\{\frac{\omega}{k} = c\right\}$$

$$\frac{|\vec{E}|}{|\vec{H}|} = \frac{\mu_0}{\sqrt{\mu_0 \varepsilon_0}} = \frac{\sqrt{\mu}}{\sqrt{\varepsilon_0}}$$

$$\frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{4\pi \times 10^{-7}}{\frac{1}{4\pi \times 9 \times 10^{9}}}}$$

$$\frac{|\vec{E}|}{|\vec{H}|} = 376.72 \text{ ohm}$$

$$\boxed{\frac{E}{H} = 376.72 \ \Omega}$$

propagation of EM wave in conducting medium

Imp In conducting medium the charge volume density is zero and current density define by ohm's law.

i.e. $\rho = 0$, $\vec{J} = \sigma \vec{E}$

then maxwell Eq^n in conducting medium is define as-

$$\nabla \cdot \vec{E} = 0 \quad (1)$$
$$\nabla \cdot \vec{H} = 0 \quad (2)$$
$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (3)$$
$$\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E} \quad (4)$$

Taking curl both side in eq^n (3)

$$\nabla \times \nabla \times \vec{E} = -\mu \cdot \frac{\partial}{\partial t} \nabla \times \vec{H}$$

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$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t}\left(\sigma \vec{E} + \frac{\varepsilon \partial \vec{E}}{\partial t}\right)$$

$$\left\{ \vec{A} \times \vec{B} \times \vec{C} = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B}) \right\}$$

$$\underbrace{\nabla(\nabla\cdot\vec{E})}_{0} - \nabla^2 E = -\mu\sigma \frac{\partial E}{\partial t} + \mu \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2 E - \mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0} \quad -(5)$$

similarly taking curl in eq^n (4) both sides -

$$\boxed{\nabla^2 H - \mu\sigma \frac{\partial \vec{H}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0} \quad -(6)$$

The solution of eq^n (5) & eq^n (6) are given by -

$$\vec{E} = \vec{E_0}\, e^{i(\vec{k}\cdot\vec{x} - \omega t)} \quad -(7)$$

$$\vec{H} = \vec{H_0}\, e^{i(\vec{k}\cdot\vec{x} - \omega t)} \quad -(8)$$

From eq^n (5) & (7) we get -

$$\nabla^2 \vec{E_0} e^{i(\vec{k}\cdot\vec{x} - \omega t)} - \mu\sigma \frac{\partial}{\partial t} E_0 e^{i(\vec{k}\cdot\vec{x} - \omega t)} - \mu\varepsilon \frac{\partial^2}{\partial t^2} \vec{E_0} e^{i(\vec{k}\cdot\vec{x} - \omega t)} = 0$$

$$[-k^2 + i\omega\mu\sigma + \omega^2 \mu\varepsilon]\, \vec{E} = 0$$

$$-k^2 + i\omega\mu\sigma + \omega^2\mu\varepsilon = 0$$

$$k^2 = \omega^2\mu\varepsilon + i\omega\mu\sigma \quad -(9)$$

$$\boxed{k = \omega\sqrt{\mu\varepsilon\left(1 + \frac{i\sigma}{\omega\varepsilon}\right)}} \quad -(10)$$

Let k is a complex function say -

$$k = A + iB \quad -(11)$$

$$k^2 = A^2 - B^2 + 2iAB \quad -(12)$$

From eq^n (9) & (12) we get -

$$A^2 - B^2 = \omega^2 \mu \varepsilon \quad -(13)$$

$$2AB = \mu \omega \sigma \quad -(14)$$

The sol^n eq^n (13) & (14) is given by -

$$A = \omega\sqrt{\mu\varepsilon}\left[\frac{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1}{2}\right]^{1/2} \quad -(15)$$

$$B = \omega\sqrt{\mu\varepsilon}\left[\frac{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1}{2}\right] \quad -(16)$$

For good conductor $\frac{\sigma}{\omega\varepsilon} >>>> 1$

$$A = \omega\sqrt{\mu\varepsilon}\left[\frac{\frac{\sigma}{\omega\varepsilon}}{2}\right]^{1/2} = B$$

$$A = \omega\sqrt{\mu\varepsilon}\left[\frac{\sigma}{2\omega\varepsilon}\right]^{1/2}$$

$$\boxed{A = \left[\frac{\omega\sigma\mu}{2}\right]^{1/2} = B} \quad -(17)$$

then

$$\vec{E} = \vec{E_0}\, e^{-\vec{B}\cdot\vec{r}}\, e^{i(\vec{A}\cdot\vec{r} - \omega t)} \quad -(18)$$

$$\vec{H} = \vec{H_0}\, e^{-\vec{B}\cdot\vec{r}}\, e^{i(\vec{A}\cdot\vec{r} - \omega t)} \quad -(19)$$

The velocity of E.m. wave in conducting medium is given by-

$$\vartheta = \frac{\omega}{K} = \frac{\omega}{A}$$

$$\vartheta = \frac{\omega}{\omega\sqrt{\mu\varepsilon}}\left(\frac{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1}{2}\right)^{1/2}$$

$$\boxed{\vartheta = \frac{1}{\sqrt{\mu\varepsilon}}\left(\frac{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1}{2}\right)^{-1/2}} \quad -(20)$$

2-mark

Skin depth and skin effect- The distance (depth) travel by electromagnatic wave in conducting medium to reduce its amptitude $\frac{1}{e}$ time the amptitude ~~and~~ at the surface of the conductor is called skin depth.

$$\boxed{\delta = \frac{1}{B} = \left(\frac{2}{\omega\sigma\mu}\right)^{1/2}}$$

From Here, it is clear that skin depth will be decreases on increasing frequency this is known as skin effect. Thus the long transmission of electricity take place at high frequency to minimize the power loss.

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V. Imp

Q. If the earth receives 2 cal / min cm^2 solar energy what are the amptitude of electric and magnatic fields of radiations.

Sol^n–

$\vec{S} = \vec{E} \times \vec{H}$

$|\vec{S}| = EH = \frac{2 \times 4.2}{60 \times 10^{-4}}$

$\boxed{EH = 1400}$ –(1)

we know that –

$\frac{E}{H} = 376.72$ –(2)

eq(1) × eq(2)

$EH \times \frac{E}{H} = 1400 \times 376.72 = 527408$

$\boxed{E = 726.22 \text{ V/m}}$

Now,

$E_0 = \sqrt{2} E$

$E_0 = \sqrt{2} \times 726.22$

$\boxed{E_0 = 1027.03}$ V/m

Now From eq^n (2)

$\frac{E}{H} = 376.72$

$H = \frac{E}{376.72} = \frac{726.22}{376.72}$

$\boxed{H = 1.92 \text{ AT/m}}$

Now, $H_0 = \sqrt{2} H$

$\boxed{H_0 = \sqrt{2} \times 1.92 = 2.71 \text{ AT/m}}$

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Assuming that all the energy from a 1000 ~~wavelong~~ watt lamp is rediated unifomly calculate the Avg. values of the intencity of electrate and magnatic field of radition at the distance 2m from lamp.

$$|\vec{S}| = EH = \frac{1000}{4\pi} \; J/sec\text{-}m^2$$

$$EH = \frac{1000}{4\pi(2)^2} = \frac{1000}{16\pi}$$

$$EH = 19.89 \quad -(1)$$

$$\frac{E}{H} = 376.72 \quad -(2)$$

Eq (1) x Eq (2)

$$EH \times \frac{E}{H} = 19.89 \times 376.72$$

$$E^2 = 7492.9608$$

$$\boxed{E = 86.59 \; V/m}$$

From eq^{4} (2) -

$$\frac{E}{H} = 376.72$$

$$H = \frac{E}{376.72} = \frac{86.59}{376.72}$$

$$\boxed{H = 0.23 \; AT/m}$$

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Que. The sun light is strike the upper atmosphere of earth with energy flux 1.38 kwatt m^{-2}. what will be the peak value of electric and magnatic fields at the point.

Aw. $\vec{S} = \vec{E} \times \vec{H}$

$|\vec{S}| = EH = 1.38 \times 10^3 \ J \ sec^{-1} m^{-2}$

$EH = 1.38 \times 10^3$ — (1)

we know that

$$\frac{E}{H} = 376.72 \quad -(2)$$

Eq (1) × Eq (2)

$E^2 = 1.38 \times 10^3 \times 376.72$

$E^2 = 519873.6$

$$\boxed{E = 721.022 \ N/m}$$

$E_0 = \sqrt{2} E$

$E_0 = \sqrt{2} \times 721.022$

$$\boxed{E_0 = 1019.68 \ V/m}$$

From eq^n (2) we get-

$$H = \frac{E}{376.72} = \frac{721.022}{376.72}$$

$$\boxed{H = 1.91 \ AT/m}$$

$H_0 = \sqrt{2} H = \sqrt{2} \times 1.91 = 2.705 \ AT/m$

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$$B_0 = \mu_0 H_0$$

$$B_0 = 4 \times 3.14 \times 10^{-7} \times 2.705$$

$$\boxed{B = 3.4 \times 10^{-6} \ wb/m^2}$$

Que The max. Electric field in a plane electro magnatic wave 10^2 N/C. The wave is going in the x- direction and Electric field in y- direction find max. magnatic field in wave and its direction.

Ans.

$$\frac{E_0}{H_0} = \mu_0 C$$

$$\frac{E_0}{\mu_0 H_0} = C$$

$$\frac{E_0}{B_0} = C$$

$$B_0 = \frac{E_0}{C} = \frac{10^2}{3 \times 10^8} = 3.33 \times 10^{-7} \ \text{Tesla}$$

Along z- direction

Que Using maxwell eq^n curl $\vec{B} = \mu_0 \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$ prove that div $\vec{D} = \rho$

Ans. Taking div of both sides -

$$\nabla \cdot \nabla \times \vec{B} = \mu_0 \left(\nabla \cdot \vec{J} + \frac{\partial}{\partial t} \nabla \cdot \vec{D}\right)$$

$$0 = \mu_0 \left(\nabla \cdot \vec{J} + \frac{\partial}{\partial t} \nabla \cdot \vec{D}\right)$$

$$\frac{\partial}{\partial t} \nabla \cdot \vec{D} = - \nabla \cdot \vec{J}$$

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$$\left\{ \nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \right\}$$

$$\frac{\partial}{\partial t} \nabla \cdot \vec{D} = \frac{\partial \rho}{\partial t}$$

$$\boxed{\nabla \cdot \vec{D} = \rho}$$

proved

Calculate the skin depth of at the Frequency 71.6 MHz in Aluminum the related parameter for aluminum are $\mu = \mu_0$

$\mu = \mu_0 = 4\pi \times 10^{-7}$ N/Amp²

$\sigma = 3.54 \times 10^{7}$ S/m

We know that -

$$\delta = \left(\frac{2}{\omega \sigma \mu}\right)^{1/2} \quad \left\{ \mu_r = \frac{\mu}{\mu_0} \right\}$$

$$\delta = \left(\frac{2}{2\pi f \sigma \mu}\right)^{1/2}$$

$$\delta = \left(\frac{1}{3.14 \times 71.6 \times 10^{6} \times 3.54 \times 10^{7} \times 4 \times 3.14 \times 10^{-7}}\right)^{1/2}$$

$\delta = 10 \times 10^{-6}$ m

$$\boxed{\delta = 10\ \mu m}$$

UNIT-3

Wave Optics

Coherent source- The light sources of same intensity, same wavelength, exactly same amplitude and zero(0) or constant phase difference are known as coherant sources.

Two independent light sources never be coherent ->
Light is produce due to the deexiteson of billion of atoms from exited state to lower energy state by emitting photon. The phase of the emitted photon change Randomly at a point so the phase difference at any point is not zero(0) or const. Thus, Two independent light source never be coherent.

Interferencs- when light from two or more then two coherant light sources propgate along the same direction then they super impose constructevely at certain point which are known as constructive interference and at other places they interfere respectivly. which are known as perseetive interference.

The modification of in the intensity of resultant light when two coherant light propgate along the same direction is called interference.

Conditions for sustain interferences - The condition for sustain interferens are as follows-

1. The intencity of light source must be same.
2. The wavelength of the source should be same,
3. The amptitude of the light source should be same
4. The phase difference b/w sources should be zero or const.
5. The sources should be narrow.
6. The light from the sources must be propgate along the same direction.
7. The sepration b/w the sources should be smaller.
8. The distance b/w the sources and screen should be larger.
9. If the interferenes accurs b/w the polarised light then the plan of polarization must be same.

Super position principle- According to super position principle the displacement of resultant light is the sum of displacement due to individual wave:

i.e.

$$y = y_1 + y_2$$

$$y_1 = a_1 \sin \omega t \quad -(1)$$

$$y_2 = a_2 \sin(\omega t + \phi) \quad -(2)$$

$$y = a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$$

$\{\sin(A+B) = \sin A \cos B + \cos A \sin B\}$

$$y = a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \sin \phi \cos \omega t$$

$$y = (a_1 + a_2 \cos \phi) \sin \omega t + a_2 \sin \phi \cos \omega t \quad -(3)$$

Let

$$a_1 + a_2 \cos \phi = A \cos \delta \quad -(4)$$

$$a_2 \sin \phi = A \sin \delta \quad -(5)$$

From eq^n 3, 4, & (5)

$$y = A \cos \delta \sin \omega t + A \sin \delta \cos \omega t$$

$$\boxed{y = A \sin(\omega t + \delta)} \quad -(6)$$

Now, eq^n $(4)^2$ + eq $(5)^2$

$$A^2 = a_1^2 + a_2^2 + 2 a_1 a_2 \cos \phi \quad -(7)$$

The intencity of wave is directly perposnal to the scqure of amplitude.

i.e.

$$I \propto A^2$$

$$I = A^2$$

$$I = a_1^2 + a_2^2 + 2 a_1 a_2 \cos \phi \quad -(8)$$

(A) Condition of maxima- The intencity will be maxima when-

$$\cos \phi = 1$$

$$\cos \phi = \cos(2n\pi)$$

so $\phi = 2n\pi$

$n = 0, 1, 2 \ldots$

path diff $(\Delta) = \frac{d}{2\pi}$ phase diff.

$$\Delta = \frac{d}{2\pi} \times 2n\pi$$

$$\boxed{\Delta = n d}$$ max.

$n = 0, 1, 2 \ldots$

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2 \quad -(9)$$

$$I_{max} = (a_1 + a_2)^2 \quad -(10)$$

(B) For minima - The intencity will be minimum when

$$\cos\phi = -1$$
$$\cos\phi = \cos(2n \pm 1)\pi$$
$$\phi = (2n \pm 1)\pi$$

$n = 0, 1, 2 \ldots$

then path diff.

$$\Delta = \frac{d}{2\pi} \times (2n \pm 1)\pi$$

$$\boxed{\Delta = (2n \pm 1)\frac{d}{2}}$$ minima

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2 \quad -(11)$$
$$I_{min} = (a_1 - a_2)^2 \quad -(12)$$

Interference and conservation Low of energy-

The avg. intensity of Interference pattern can be expressed as

$$I_{av} = \frac{I_{max} + I_{min}}{2}$$

$$I_{av} = \frac{a_1^2 + a_2^2 + 2a_1a_2 + a_1^2 + a_2^2 - 2a_1a_2}{2}$$

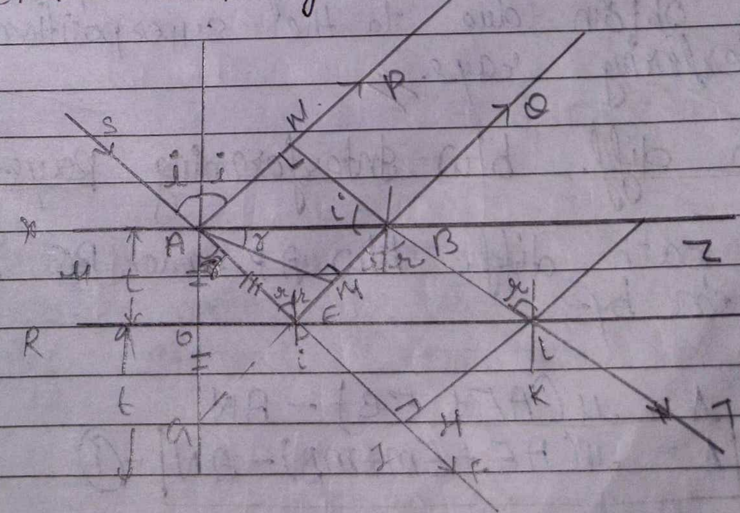
$$I_{av} = \frac{2a_1^2 + 2a_2^2}{2}$$

$$\boxed{I_{av} = a_1^2 + a_2^2}$$

Thus, the avg. intensity is the sum of the intensity of individual wave. So, Interference pattern is consistant with conservation low of energy. whatever energy may be apparently this appear at minima actully present at maxima.

Interference due to uniform their films (Parallel the film) -

95% most Imp



when monochromatic light incident on the upper surface of uniform thin film then interference pattern due to the superposition of interfering range occurs.

(I) In Reflected system
(II) In transmitted system

Due to the division of amplitude.

Let's consider a thin film of refractive index (μ) and thickness (t) is bounded between the uper surface (X-Y) and Lower surface (R-Z).

when monochromatic light incident at a on the upper surface then it suffer multiple reflection and refraction from upper and lower surface of the film and interference pattern is obtain due to the superposition of Interfering rays.

path diff. b/w Interfering rays-

The path diff. b/w AP and BQ is given by-

$$\Delta = \mu(AE + EB) - AN$$

$$\boxed{\Delta = \mu(AE + Em + mB) - AN} \quad -(1)$$

using Snell's law-

$$\frac{\sin i}{\sin r} = \mu = \frac{AN/AB}{mB/AB}$$

$AN = \mu mB$ put eq^n (1)

$$\Delta = \mu(AE + Em) + \mu mB - \mu mB$$

$$\boxed{\Delta = \mu(AE + Em)} \quad -(2)$$

Now,

$$\Delta = \mu(GE + Em) \quad \left\{ \begin{array}{l} \Delta AEO \cong \Delta GEO \\ AE = GE \end{array} \right.$$

$$\boxed{\Delta = \mu Gm} \quad -(3)$$

in Δ AGm

$$\cos r = \frac{Gm}{AG}$$

$$Gm = AG \cos r$$

$$Gm = (AO + OG)\cos r \quad \left\{ \begin{array}{l} \Delta AEO \cong \Delta G \\ AO = GO = t \end{array} \right.$$

$$Gm = (t + t)\cos r$$

$Gm = 2t\cos r$ put eq^n (3)

$$\boxed{\Delta = 2\mu t \cos r} \quad -(4)$$

(A) Interference in reflected system- The total path diff. in reflected system using Stroke's treatment is given

$$\Delta T = \Delta - \frac{\lambda}{2}$$

$$\Delta T = 2\mu t \cos r - \frac{\lambda}{2}$$

(i) Condition for maxima- The intencity will be maximum -

$$\Delta_T = n\lambda$$

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$\boxed{2\mu t \cos r = (2n+1)\frac{\lambda}{2}}$$

(ii) Condition for minima- The intencity will be minima when-

$$\Delta_T = (2n+1)\frac{\lambda}{2}$$

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda - \frac{\lambda}{2}$$

$$\boxed{2\mu t \cos r = n\lambda}$$

(B) Interferance in transmitted system-

The total path diff. in transmitted system-

$$\boxed{\Delta_T = 2\mu t \cos r}$$

(i) Condition for maxima- The intencity will be maxima-

$$\boxed{2\mu t \cos r = n\lambda}$$

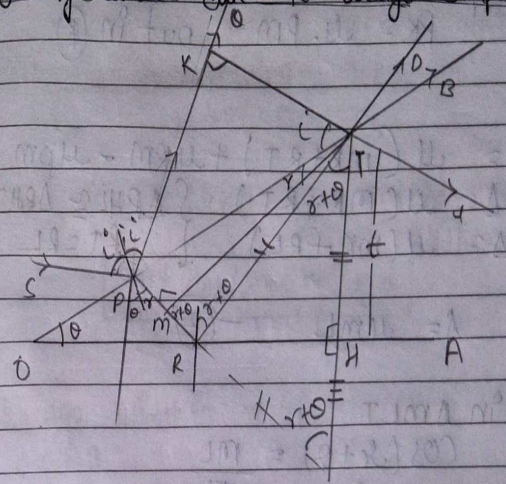
Condition for maxima- The intencity will be minima-

$$\Delta_T = (2n+1)\lambda/2$$

$$\boxed{2\mu t \cos r = (2n+1)\frac{\lambda}{2}}$$

Thus the Condition for maxima or minima are complementry with each other in reflected and transmitted system.

Interferance due to wedge shape thin film-



When a thin film refracting index μ is bounded b/w two inclined surface (OA) and (OB) of varying thickness then it is caused wedge shape thin film.

When a monochromatic light incident on the upper surface of wedge shaped film at point P then it suffer multiple reflection and refraction from the upper and lower surface of the film then the path diff. b/w interfering wave is given by-

$$\Delta = \mu(PR + RT) - PK$$

$$\Delta = \mu(PM + MR + RT) - PK \quad (1)$$

Using Snells law-

$$\frac{\sin i}{\sin r} = \mu = \frac{PK/PI}{PM/PT}$$

$PK = \mu . PM$ put in (1)

$$\Delta = \mu(MR+RT) + \mu PM - \mu PM$$
$$\Delta = \mu(MR+RT) \quad \left\{ \begin{array}{l} \Delta RHL \cong \Delta RHT \\ RT = RL \end{array} \right\}$$
$$\Delta = \mu(MR+RL)$$

$$\Delta = \mu ML \quad -(2)$$

in ΔMLT

$$\cos(r+\theta) = \frac{ML}{TL}$$

$$ML = TL \cos(r+\theta)$$
$$ML = (TH+HL)\cos(r+\theta)$$
$$ML = (t+t)\cos(r+\theta) \quad \left\{ \begin{array}{l} \Delta RHL \cong \Delta RHT \\ TH = HL = t \end{array} \right.$$
$$ML = 2t\cos(r+\theta)$$

put in eq^n (2) we get

$$\boxed{\Delta = 2\mu t\cos(r+\theta)}$$

Interferance in reflected system-

$$\Delta_T = \Delta - \frac{d}{2}$$

$$\Delta_T = 2\mu t\cos(r+\theta) - \frac{d}{2}$$

(1) Condition for maxima-

$$\Delta_T = nd$$
$$2\mu t\cos(r+\theta) - \frac{d}{2} = nd$$

$$2\mu t\cos(r+\theta) = (2n+1)\frac{d}{2}$$

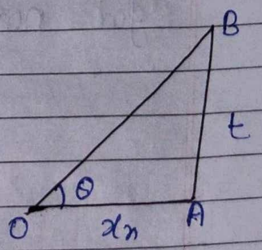
(2) Condition for minima-

$$\Delta_T = nd - \frac{d}{2}$$

$$2\mu t\cos(r+\theta) - \frac{d}{2} = nd - \frac{d}{2}$$

$$\boxed{2\mu t\cos(r+\theta) = nd}$$

Fringe width-



ΔOAB

$$\tan\theta = \frac{P}{B} = \frac{t}{x^n}$$

$$t = x^n \tan\theta$$

The condition for minima reflected system

$$2\mu t\cos(r+\theta) = nd$$

then $2\mu x_n \tan\theta \cos(r+\theta) = nd \quad -(1)$

The Condition for $(n-1)^{th}$ minima in reflected system-

$$2\mu t x_{n-1} \tan\theta\cos(r+\theta) = (n-1)d \quad -(2)$$

Now, (1) - (2) we have

$$2\mu \tan\theta\cos(r+\theta)[x_n - x_{n-1}] = nd - nd + d$$

The circular concentrate interferance frange are observed when a thin film of air or other transparent medium is enclosed b/w plane glass plate and plano convex lens such franges are known as Newton's Ring.

When a monochromatic light incident normally on the upper surface of plano convax lens. Then the Ray reflected from upper and lower surface of film in such a way that it produce circular rings.

Newton's Ring a circular because the locus of constant thickness about the point of contact circular.
The centre of Newtons Rings in reflected system is Dark because path diff. in reflected system

$$\Delta = 2\mu t - \frac{\lambda}{2}$$

$$t = 0$$

$$\Delta = -\frac{\lambda}{2}$$

which is the condition of minima.

So, centre becomes dark.

Let CME be vertical section of plano convex lens.

$$CD \times DE = AD \times DM$$

$$CD \times DE = (AM - DM) \times DM$$

Let r_n and d_n be the radious and diameter of n^{th} ring passes through C and E.

$$r_n \times r_n = (2R - t) \times t$$

$$r_n^2 = 2Rt - t^2$$

$$r_n^2 = 2Rt \qquad \left\{ \begin{array}{l} t <<<< \\ \text{then } t^2 \simeq 0 \end{array} \right.$$

$$2t = \frac{r_n^2}{R}$$

$$\boxed{2t = \frac{d_n^2}{4R}} \qquad \left\{ r_n = \frac{d_n}{2} \right\}$$

The path difference b/w interfering Ray is given by-
when $\theta <<<$ for normal incidence

(A) Interferance in reflected system - The total path difference in reflected system ~~The~~ ~~to~~ using Stokes treatment is given

$$\Delta_T = 2\mu t - \lambda/2$$

(A) for maxima-

$$\Delta_T = n\lambda$$

$$2\mu t - \frac{\lambda}{2} = n\lambda$$

$$2\mu t = \left(n + \frac{1}{2}\right)\lambda$$

for air medium $\boxed{\mu = 1}$

$$2t = \left(n+\frac{1}{2}\right)\lambda$$

$$\frac{d_n^2}{4R} = \left(n+\frac{1}{2}\right)\lambda$$

$$d_n^2 = 4R\lambda\left(\frac{n+1}{2}\right)$$

$$d_n \propto \sqrt{\left(n+\frac{1}{2}\right)} \quad \text{Bright}$$

Thus the diameter of n^{th} bright ring in reflected system is directly perportional to squre root of half of odd tive integer.

(ii) for minima- Intensity will be minimum

$$2\mu t - \frac{\lambda}{2} = n\lambda - \frac{\lambda}{2}$$

$$\boxed{2\mu t = n\lambda}$$

for air $\mu = 1$

$$2t = n\lambda$$

$$\frac{d_n^2}{4R} = n\lambda$$

$$\boxed{d_n^2 = 4R\lambda n}$$

$$\boxed{d_n \propto \sqrt{n}} \quad \text{Dark}$$

Thus the diameter of n^{th} dark ring in reflected system is directly perpotinal to squre root of positive integer.

(B) Interferance in transmitted system- The path diff. in transmitted system is given by

$$\boxed{\Delta_T = 2\mu t}$$

(I) Condition for maxima-

$$\Delta_T = n\lambda$$

$$2\mu t = n\lambda$$

for air $\mu = 1$

$$2t = n\lambda$$

$$\frac{d_n^2}{4R} = n\lambda$$

$$d_n^2 = 4Rn\lambda$$

$$\boxed{d_n \propto \sqrt{n}} \quad \text{Bright}$$

(II) for minima-

$$2\mu t = \left(n+\frac{1}{2}\right)\lambda$$

for air $\boxed{\mu = 1}$

$$2t = \left(\frac{n+1}{2}\right)\lambda$$

$$\frac{d_n^2}{4R} = \left(\frac{n+1}{2}\right)\lambda$$

$$\boxed{d_n \propto \sqrt{n+\frac{1}{2}}} \quad \text{Dark}$$

Determination of wavelength using Newton Ring- The condition for n^{th} minima in reflected system for air medium is given by -

$$d_n^2 = 4Rd\,n \quad -(1)$$

Condition for m^{th} minima in ref. sys. for air medium is given by

$$d_m^2 = 4mRd \quad -(2)$$

Now, (2) - (1)

$$d_m^2 - d_n^2 = 4Rd(m-n)$$

Let $m-n=P$

$$\boxed{m = n+P}$$

$$d^2_{n+p} - d^2_n = \cancel{4Rn}\ 4PRd$$

$$\boxed{d = \frac{d^2_{n+p} - d^2_n}{4PR}}$$

determination of refractive index-

The condition for n^{th} minima in reflected system in a medium of refractive index μ is given by:-

$$\mu\, d^2_{n\,liquid} = 4nRd \quad -(1)$$

The condition for mth minima in reflected system for air medium is given by

$$d^2_{n\,air} = 4Rnd \quad -(2)$$

eq (1) / 2

$$\frac{\mu d^2_{n\,liq}}{d^2_{n\,air}} = 1$$

$$\boxed{\mu = \frac{d^2_{n(air)}}{d^2_{n(liq)}}}$$

Q. Calculate the thickness of the thin film ($\mu = 1.4$) in which interferance of violet component ($\lambda = 4000$ Å) of incident light can take place by reflection?

Ans. $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

$$t = \frac{(2n+1)\lambda/2}{2\mu \cos r}$$

$$t_{min} = \frac{\lambda/2}{2\mu} \quad \left\{ \begin{array}{l} \text{For } t_{min} \\ n=0,\ \cos r = 1 \end{array} \right.$$

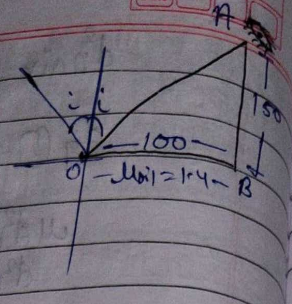
$$t_{min} = \frac{\lambda}{4\mu} = \frac{4000\,Å}{4 \times 1.4}$$

$$\boxed{t_{min} = 714.3\ Å}$$

Q. A men whose eyes 150 cm above the coil film on water surface observe greenesh colour at a distance of 100 cm from his feet. Calculate the proable thickness of film.

80%

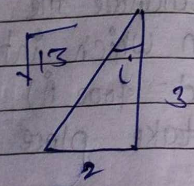
Ans. $\lambda_{green} = 5000\,\text{Å}$
$\mu_{oil} = 1.4$
$\mu_{water} = 1.33$



$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

$t = \frac{(2n+1)\,\lambda/2}{2\mu\cos r}$

$\Delta OAB \quad \tan i = \frac{100}{150} = \frac{2}{3}$



$\sin i = \frac{2}{\sqrt{3}}$

$\frac{\sin i}{\sin r} = \mu$

$\sin r = \frac{\sin i}{\mu}$

$\cos r = \sqrt{1 - \frac{\sin^2 i}{\mu^2}} = \sqrt{1 - \frac{4/13}{(1.4)^2}}$

$\cos r = 0.9182$

$t = \frac{5000 \times 10^{-8}}{2 \times 2 \times 1.4 \times 0.9182}(2n+1)$

$t = 9.725 \times 10^{-6}(2n+1)\,cm$ Ans.

$t_0 = 9.725 \times 10^{-6}\,cm$

Q. White light is incident on a soap film at an angle $\sin^{-1} 4/5$ and the reflected light is observed with spectroscope. It is found with two consecutive dark bands correspond to wavelength 6.1×10^{-5} cm, 6×10^{-5} cm. If the refractive index of the film $\mu = \frac{4}{3}$ calculate the thickness?

Ans. $2\mu t \cos r = n\lambda_1 \quad -①$
$2\mu t \cos r = (n+1)\lambda_2 \quad -②$

$(n+1)\lambda_2 = n\lambda_1$
$\lambda_2 = n\lambda_1 - n\lambda_2$
$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$

$2\mu t \cos r = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$

$t = \frac{\lambda_1 \lambda_2}{2\mu \cos r(\lambda_1 - \lambda_2)}$

$\cos r = \sqrt{1 - \left(\frac{\sin^2 i}{\mu^2}\right)}$

$\cos r = \sqrt{1 - \frac{(4/5)^2}{(4/3)^2}} = \frac{4}{5}$

$t = \frac{6.1 \times 10^{-5} \times 60 \times 10^{-5}}{2 \times \frac{4}{3} \times \frac{4}{5} \times 0.1 \times 10^{-5}} = 0.0017\,cm$

Q. Imp Light of wavelength 600 Å falls normally on a thin wedge shaped film of refractive index 1.4 forming fringes that are 2.0 mm apart find the angle of wedge in seconds.

Sol^n.

$\lambda = 6000 Å$

$\mu = 1.4$

$\beta = 2.0$ mm

$\beta = \frac{\lambda}{2\mu\theta}$

$\theta = \frac{\lambda}{2\mu\beta}$ rad.

$\theta = \frac{\lambda}{2\mu\beta} \times \frac{180}{\pi} \times 60\times60$ sec

$\theta = \frac{6000\times10^{-10}}{2\times1.4\times2.0\times10^{-3}} \times \frac{180}{\pi} \times 60\times60$

$\theta = 22.09$ sec

Q. Imp Newtons Ring are observed normally in reflected light of wavelength 6000 Å the diameter of a 10^{th} dark ring is 0.50 cm find the radius of curvature of the lens and the thickness of film.

Ans.

$\lambda = 6000 Å$

$d_{10} = 0.50$ cm

$2\mu t = n\lambda$

for air $\mu = 1$

$2t = n\lambda$

$\frac{d_n^2}{4R} = n\lambda$

$R = \frac{d_n^2}{4n\lambda}$

$R = \frac{(0.50)^2}{4\times10\times6000\times10^{-8}}$

$R = 104$ cm

(ii)

$2t = \frac{d_n^2}{4R}$

$t = \frac{d_n^2}{8R} = \frac{(0.50)^2}{8\times104}$

$\boxed{t = 3\times10^{-4}\ cm}$

Q. Newtons Ring are observed by keeping a spherical surface of 100 cm Radius on a plane glass plate the diameter of 15 bright Ring is 0.590 cm and the diameter of 5^{th} Ring is 0.336 cm what is the wavelength light is used.

Ans.

$\lambda = \frac{d_{n+p}^2 - d_n^2}{4PR}$

$R = 100$ cm

$d_{15} = 0.590$ cm

$d_5 = 0.336$ cm

$\lambda = ?$

$\lambda = \frac{d_{15}^2 - d_5^2}{4\times P\times R} = \frac{(0.59)^2 - (0.336)^2}{4\times10\times100}$

$\left\{ \begin{matrix} P = 15-5 \\ P = 10 \end{matrix} \right\}$

$\boxed{\lambda = 5880\ Å}$

Imp

Q. In Newtons Ring experiment the diameter of the 4th & 12th dark ring are 0.400 & 0.700 cm respectively deduce the diameter of the 20th dark ring

Sol^n

$d_1 = 0.400$ cm

$d_2 = 0.700$ cm

$d_{20}^{th} = ?$

Ans. $\lambda = \frac{d^2_{n+p} - d^2_n}{4PR}$

$d^2_{n+p} - d^2_n = 4PR\lambda$ — (1)

$d^2_{12} - d^2_4 = 4 \times 8 \times R\lambda$ — (2)

$d^2_{20} - d^2_{12} = 4 \times 8 \times R\lambda$ — (3)

$\frac{Eq\ (3)}{Eq\ (2)} = \frac{d^2_{20} - d^2_{12}}{d^2_{12} - d^2_4} = 1$

$d^2_{20} = 2d^2_{12} - d^2_4$

$d^2_{20} = 2(0.700)^2 - (0.400)^2$

$d^2_{20} = 0.82$

$\boxed{d_{20} = 0.90}$